ISSN 2518-1726 (Online),
ISSN 1991-346X (Print)

ҚАЗАҚСТАН РЕСПУБЛИКАСЫ
ҰЛТТЫҚ ҒЫЛЫМ АКАДЕМИЯСЫНЫҢ
Әль-фараби атындағы Қазақ ұлттық университетінің

Х А Б А Р Л А Р Ы

ИЗВЕСТИЯ

НАЦИОНАЛЬНОЙ АКАДЕМИИ НАУК
РЕСПУБЛИКИ КАЗАХСТАН
Казахский национальный университет
имени Аль-фараби

N E W S

OF THE NATIONAL ACADEMY OFSCIENCES
OF THE REPUBLIC OF KAZAKHSTAN
Al-farabi kazakh
national university

SERIES
PHYSICO-MATHEMATICAL

1 (323)

JANUARY – FEBRUARY 2019

PUBLISHED SINCE JANUARY 1963

PUBLISHED 6 TIMES A YEAR

ALMATY, NAS RK

«ҚР ҰҒА Хабарлары. Физика-математикалық сериясы».

ISSN 2518-1726 (Online), ISSN 1991-346Х (Print)

Меншіктенуші: «Қазақстан Республикасының Ұлттық ғылым академиясы» РҚБ (Алматы қ.)
Қазақстан республикасының Мәдениет пен ақпарат министрлігінің Ақпарат және мұрағат комитетінде 01.06.2006 ж. берілген **№5543-Ж** мерзімдік басылым тіркеуіне қойылу туралы куәлік

Мерзімділігі: жылына 6 рет.
Тиражы: 300 дана.

Редакцияның мекенжайы: 050010, Алматы қ., Шевченко көш., 28, 219 бөл., 220, тел.: 272-13-19, 272-13-18, http://physics-mathematics.kz/index.php/en/archive

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Типографияның мекенжайы: «Аруна» ЖК, Алматы қ., Муратбаева көш., 75.

«Известия НАН РК. Серия физико-математическая».

ISSN 2518-1726 (Online), ISSN 1991-346X (Print)

Собственник: РОО «Национальная академия наук Республики Казахстан» (г. Алматы)
Свидетельство о постановке на учет периодического печатного издания в Комитете информации и архивов Министерства культуры и информации Республики Казахстан **№5543-Ж**, выданное 01.06.2006 г.

Периодичность: 6 раз в год.
Тираж: 300 экземпляров.

Адрес редакции: 050010, г. Алматы, ул. Шевченко, 28, ком. 219, 220, тел.: 272-13-19, 272-13-18,
http://physics-mathematics.kz/index.php/en/archive

Адрес типографии: ИП «Аруна», г. Алматы, ул. Муратбаева, 75.

News of the National Academy of Sciences of the Republic of Kazakhstan. Physical-mathematical series.

ISSN 2518-1726 (Online), ISSN 1991-346X (Print)

Owner: RPA "National Academy of Sciences of the Republic of Kazakhstan" (Almaty)
The certificate of registration of a periodic printed publication in the Committee of information and archives of the Ministry of culture and information of the Republic of Kazakhstan N 5543-Ж, issued 01.06.2006

Periodicity: 6 times a year
Circulation: 300 copies

Editorial address: 28, Shevchenko str., of. 219, 220, Almaty, 050010, tel. 272-13-19, 272-13-18,
http://physics-mathematics.kz/index.php/en/archive

Address of printing house: ST "Aruna", 75, Muratbayev str, Almaty

N E W S
OF THE NATIONAL ACADEMY OF SCIENCES OF THE REPUBLIC OF KAZAKHSTAN
PHYSICO-MATHEMATICAL SERIES
ISSN 1991-346X **https://doi.org/10.32014/2019.2518-1726.1**
Volume 1, Number 323 (2019), 5 – 13

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BUSINESS-PROCESS DEVELOPMENT OF THE INFORMATION-ANALYTICAL SYSTEMS OF THE BAIKONUR COSMODROM AND LAUNCH VEHICLE DESIGN FOR ECOLOGICAL SAFETY IMPROVINGIN THE IMPACT AREAS OF THEWORKED-OFF STAGES

Abstract. The analysis of the existing information-analytical system (IAS) of the Baikonur cosmodrome (IAS_{cd}) and the launch vehicles design (IAS_{lv}) are carried out. The main sources of the technogenic impact of LV launching with the main liquid propulsion engines in the impact areas of worked-off stages (WS) are shown. The concept of modernization of the existing IAS_{cd} and IAS_{lv} is proposed, which provides for the reduction of the technogenic impact for none rasable expendable worked-off stages, based on operational recommendations of the created IAS_{lv} andIAS_{lv}^{es} on the fire-explosion safety of the worked-off stages, reducing the size of theimpact areas of the worked-off stages, and the possibility of theworked-off stagesmaneuvering to change the impact area. Proposals for the modernization of the existing IAS_{cd} and the design concept for non-reusable LVs, based on the conditions for improving ecological safety, have been developed.

Key words: technogenic impact, falling areas, information analytical system, purposed medium rocket step, components of rocket fuel.

Introduction

The development of advanced LVs with the main liquid propulsion engines (LPE), in accordance with the accepted recommendations of such organizations as the UN's Technical Subcommittee on the Peaceful Uses of Outer Space [1], the Inter-Agency Space Debris Coordination Committee (IADC) [2] provides a significant reduction in the technogenic impact of LV launches with mainLPE on the environment, including:

- prevention of pollution of the near-Earth space by the upper WS with the main LPE which are large explosive space debris;
- a drastic reduction in the number and areas of impact areas on the surface of the Earth for the lower WSs, which are fire hazardous and toxic objects, leading to the chemical contamination of soil with residues of liquid toxic propellant components such as unsymmetrical dimethylhydrazine, nitric acid, kerosene.

Developers and operators of LV with LPE are interested in applying technologies, schematic and designsolutions aimed at increasing the ecological safety of LV to modern requirements, while the technical solutions should not worsen the achieved performance in terms of tactical and technical characteristics, reliability, use of proven technologies for the LV production, ground tests and operation.

The location of the Baikonur Cosmodrome is such that considerable areas of the impact areas of the lower WSare located on its own land territory of Russia and Kazakhstan. During the LV launches from the Baikonur cosmodrome, 28 impact areas(IA) are deployed in Russia (4.5 million hectares, including 0.12 million hectares in the Omsk Region, 0.96 million hectares in the Novosibirsk Region, 1.96 million hectares in the Tomsk region, 0.4 million hectares in the Tyumen region, 0.53 million hectares in the Altai Republic, 0.15 million hectares in the Republic of Sakha (Yakutia)), 52IAs in the Republic of Kazakhstan (4.6 million hectares), 4 IAs in the Republic of Turkmenistan (1.19 million hectares), 2 IAs in the Republic of Uzbekistan (0.17 million hectares) [3].

The impact areas of the lower WS in the USA, the European Union, Japan, India, Brazil are located in the waters of the World Ocean, therefore, the issues of ensuring ecological safety in the impact areas in comparison with Russia and Kazakhstan are virtually absent.

Of considerable interest are the works carried out in the United States on reusable lower WS, for example, launches of the rescued lower WS of LV"Falcon-9" [4], LV "Sheppard" [5], in which an attempt is made simultaneously to solve two basic problems arising in the rocket and space activities:

- reducing the cost of the payloads insertion due to the multi-use of the most expensive part of the LV with the main LPE (lower WS);
- reducing the technogenic impact of LV launches with the main LPE in the impact areas of the WS due to the return of the lower WS to the launch site, which is more important for Russia with its location of cosmodromes than for the USA.

In terms of the economic efficiency of such LVs (the ratio of the cost of the payload insertion into thespecified orbit to the cost of the total payload insertion), we can refer to the experience of operating the Space Shuttle reusable transport space system (RTSS) using technology, schematic and designand construction solutions based on a manned aerodynamic (airplane) landing scheme. Operational experience has convincingly shown that the economic efficiency of the non-reusable LV is much higher than the efficiency of RTSS [6]. The data on the economic efficiency for the LV with the main LPE using technologies, schematic and designand construction solutions applied at the Falcon-9 LV in comparison with the economic efficiency of traditional non-reusable LVs in open press have not been detected, although work has been known to analyze the effect of the flight scheme of the stage with a rocket-dynamic rescue system for the energy characteristics of a two-stage medium-range LV[7].

Similar studies are being conducted in Russia, for example, the projects "Rossiyanka" [8], "Baikal" [9], "Demonstrator" [10], using both a rocket-dynamic maneuver for the soft landing [7] and aerodynamic maneuver (airplane landing scheme type of RTSS Space Shuttle, Buran) [8, 9].

The shortcomings of the technologies, schematic and designand construction solutions used in the above developments are significant losses in the payload mass, complex technical solutions that lead to the large volumes of ground testing and, accordingly, the high cost of the LV launch due to its multi-use [6].

The study [11, 12] formulated the main factors of the technogenic impact of the LV launches with the main LPE in the impact areas of lower WS and conceptual proposals for their cardinal reduction. Thesemainfactorsinclude:

- unused liquid propellant residues components in tanksof the WS after the mainLPE cut-off, which entails an increased probability of explosion of the fuel tanks both at the atmospheric section of the WS descent trajectory, and directly on the surface of the impact area, increasing the probability of fire hazard of vegetation cover inflammability;
- the presence of uncontrolled motion of the WS in the atmospheric section of the WS descent trajectory, which leads to a significant dispersion of the points of fall of the WS and its fragments, respectively, of the area of the impact areas with a probability of 10^{-4} of the non-reflection of a man [2].

Taking into account the conducted analysis, it is proposed to consider the concept of ensuring ecological safety (ES) based on the following postulates:

P1. The life cycle of the WS should not end, as it is implemented at the present time in the logic of the functioning of virtually all Russian launch vehicles launched from the Baikonur cosmodrome - achieving specified movement parameters, cutting-off the main LPE. Should still be implemented phase of the WS operating, by analogy with the spacecraft, providing for its transfer to the utilization orbit after the end of the active life. At this phase, the WS should ensure minimization of technogenic impact on the environment in the area of its expected fall.

P2. At the present stage of the study, it is not supposed to return the WS to the cosmodrome with its soft landing and subsequent reuse, similar to the first WS of the Falcon-9 LV.

P3. Ideal option - the fall of the WS with almost "dry" fuel tanks and fuel lines with a minimum deviation from the projected point of fall of the WS, located in the R-neighborhood from the energy-optimal point of fall of the WS.

Implementation of this concept involves:

1. The presence in the information and analytical system of IAS_{ia}, which is the part of the general IAS of the cosmodrome IAS_{cd} [13, 14], information on the ecological consequences of the WSfall to the initial predicted point of fall selected by the developer and operator of the launch vehicle, including:

a) meteorological conditions in the neighborhood of point of fall,

b) prediction of the possibility of vegetation fire taking into account climatic and meteorological conditions,

c) the spread of the vapor cloud of the fuel component,

d) alternative points of fall of the WS with the corresponding characteristics, etc. the above information must be generated

This information from the IAS_{ia} is necessary to make a decision by the LV developer for the purpose of developing technologies, schematic and design and construction solutionsfor improving the ecological safety of the LV in the impact area.

2. The presence in the IAS_{ia}information and analytical system, which is the part of the overall system for the design and exploitation of LV, the following information:

a) the possibility of changing the predicted coordinates of the point of fall of WSto the other recommended points in the impact area, where the ecological consequences due to the characteristics of the impact area [15, 16] will be significantly less;

b) options for changing the coordinates of the points of fall of the WS, for example, by changing the pitch program, yawing on the active section of the LVlaunching phase, [17] or by an additional autonomous on-board descent system (ABDS) installed on the WS [18], to implement the WS maneuver into other possible points of fall in the same designated impact area, but with more acceptable characteristics;

c) use of the energy optimal pitch program and the corresponding predicted optimal point of fall of the WS, while this point of fall must be in the R-neighborhood from the energy-optimal point of fall of the WS; The R-neighborhood is determined by the energy capabilities of the ABDS, the time of passive WS flight from the moment of separation from the LV to the moment of contact of the surface of the impact area.

In addition to the information received from the IAS_{ia}, which is necessary to improve the ecological safety of LV, the IAS_{lv}works on:

a) minimization of fuel residues in tanks after cutting-off of the main liquid propulsion engine;

b) assessment of the ABDS ballistic capabilities for the WS maneuvering on the trajectory of descent;

c) estimation of the possible spillages of residual fuel components from collapsed fuel tanks and WS lines in the predicted WS point of fall;

d) probability estimation of the WS explosion and the expected zone of fragment dispersion, etc.

1 Statement of the research problem

In accordance with the above analysis and the formulated concept, the general problem of improving the ecological safety of the LV with mainLPE can be decomposed into three interrelated sub problems:

- development of IAS_{ia} as a component of the IAS_{cd}, determination of a list of additional tasks, mathematical models and software products that implement them;

- development of IAS_{lv}^{es}, as an integral part of the IAS_{ia} of the existing system of design and operation of LV, determination of a list of additional tasks, technologies, schemes and design solutions aimed at improving the ecological safety of LV;

- determination of the optimal interaction, information flows between the IAS_{ia} andIAS_{lv}^{es}, the criterion of optimality and boundary conditions.

2 Development of IAS_{ia}

The system of ecological monitoring of Baikonur Cosmodrome (SEMC) conceptually includes three main systems: the information-analytical system, the geo-information system and the monitoring system.

A number of works have been devoted to various aspects of the construction of such systems, for example, [14-16], in which the system of ecological monitoring of the Baikonur Cosmodrome was considered as part of the overall monitoring system, which it was possible to distinguish the component of the technogenic impact of the rocket and space activities on the environment. Figure 1 shows the general structure of the ecological monitoring of the Baikonur cosmodrome.

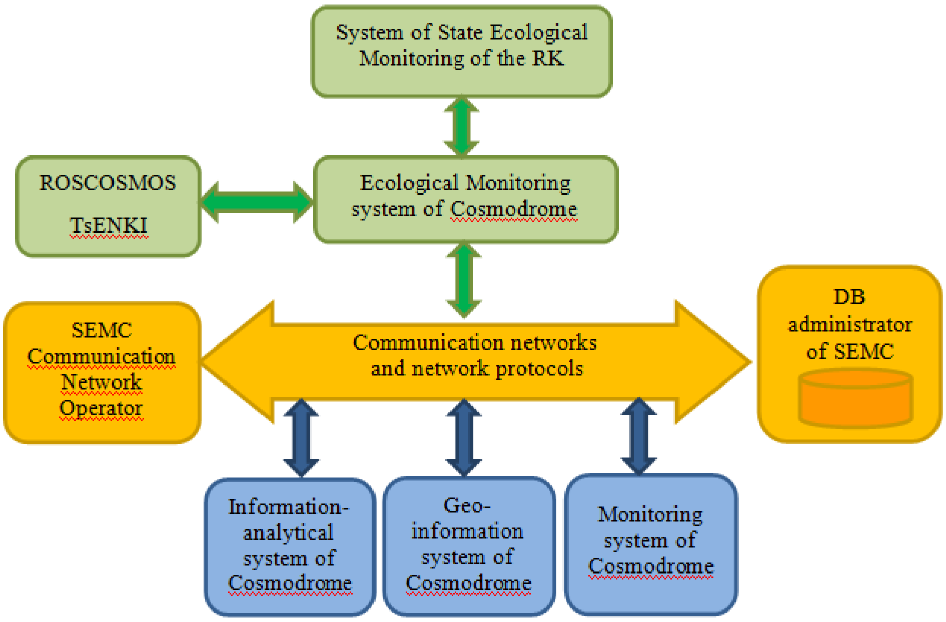

Figure 1 - System of ecological monitoring of Baikonur cosmodrome activity

The proposed approach is based on the separation of the IAS_{cd} functions into two parts: the basic IAS_{cdb} and IAS_{ia}.

The task of the IAS_{cdb} includes traditional assessments of the ecological monitoring of the Baikonur cosmodrome, based on obtaining information from the materials of the ecologicalcertification of the impact areas of the WS in accordance with the passport of the IA [15, 16, 19]. Theseassessmentsinclude:

- general information about the enterprise responsible for the operation of the IA;
- general information about the impact area of the WS and adjacent territories;
- characteristics of natural and climatic conditions in the impact area territory;
- information on economic activities in the impact area and in adjacent territories;
- characteristics of pollution sources of the IA, etc .;
- Calculation of ecological damage $E_i\,[\vec{R}_i(x_i, y_i)]$ and the cost of restoration work $C_i^{rw}[\vec{R}_i(x_i, y_i)]$ for each launch.

The task of IAS_{ia} includes:

a) from the received data on the upcoming LV launch from the IAS_{lv} (the initial aiming point of the WS fall in the assigned IA $\vec{R}^{in}_{aim}(x_i, y_i)$, the optimal aiming point at which the payload mass inserted to the specified orbit is maximal, dividing the area of the IA by N sections with Si areas (i = 1, ... N), so that $\sum_{i=1}^{N} S_i = S_\Sigma$;

b) in the chosen N areas, N possible predictable coordinates of the points of fall of the WS are selected;

c) distances $\Delta\vec{R}_i = \vec{R}_{opt}(x, y) - \vec{R}_{pr}(x_i, y_i)$ are estimated for assessing the possibility of WS maneuvering by shifting the point of fall of the WS to these values and transmitted to the IAS_{lv};

d) on the basis of the passport of this IA, the ecological damage $E_i\ [\vec{R}_i(x_i, y_i)]$ from falling into this i-th section and, accordingly, the cost of restoration works, is calculatedthe each predicted point of fall $\vec{R}_i(x_i, y_i)$;

d) the received information is transmitted to the IAS_{lv} for the calculation of the LV movement control programs in the active section of the launch trajectory and the WS control programs in the descent section to the selected point, which is determined from the analysis of the data array $\{C_i^{sa}[\vec{R}_i(x_i, y_i)]\}$, the estimation of the ballistic capabilities of the ABDS for maneuvering by changing the coordinates of the point of the fall by a $\Delta\vec{R}_i = \vec{R}_{opt}(x, y) - \vec{R}_{pr}(x_i, y_i)$ value.

In Fig. 2, as an example, the impact area for the Proton LV is given.

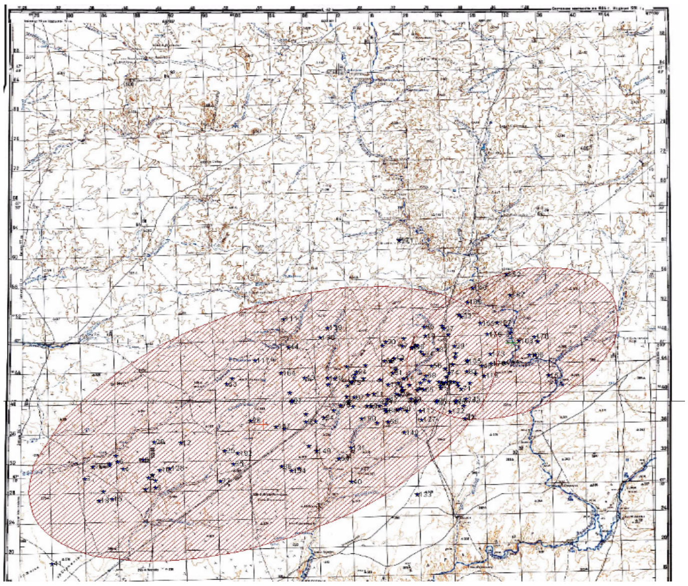

Figure 2 - The impact area for the "Proton" LV

As follows from Fig. 2, it is possible to ensure the fall of the WS into areas with significantly different landscape conditions. At the same time, it is assumed that an ABDS is installed on the WS, which provides control of the WS movement on the descent trajectory. As a result of this control, the accuracy of the WS fall is similar to the landing accuracy of the Falcon-9 LV WS when landing at a cosmodrome or a floating barge.

3 IAS_{lv}^{es} development

IAS_{lv}^{es} is an element of the existing IAS_{lv}, which includes information and analytical models of the LV, starting with the stage of formation of tactical and technical and technical tasks, including the choice of design and construction parameters for LV, design and construction, technological, production documentation including for testing at all stages of fabrication of the material part in the manufacturer), operational documentation (for work on the technical and launch complexes of LV), network schedules of the work plan for the various phases of the LVlife cycle.

As noted above, the life cycle and, accordingly, the technological,schematic and design and construction solutionsfor LV, equipment for LV testing and checking are oriented to complete the cycle with a command to cut off the main LPE after achieving specified movement parameters and the payloadseparation. Further operation of the WS is a different kind of risks: explosion in theorbit, collision with other orbital objects before fires and pollution by WS fragments in the impact areas [1-3, 19].

IAS_{lv}^{es} development within the framework of the concept of improving the ecological safety of LV with main LPE in the impact areas of the WS provides for the use of information from the IAS_{ia} in several areas:

a) to change the program for controlling the movement of the launch vehicle at the launchingphase (changing the points of aiming for the WS fall: the optimum point, an acceptable point from the condition of minimizing ecological damage, which is achieved by the adjusting the existing techniques for calculating LV launch programs);

b) to develop control of the WS movement with the ABDSuse while moving along the trajectory of descent to the selected point on the territory of the WS impact area;

c) for the ABDS creation, which requires to complete a full cycle of its development with the assessment of the impact of the ABDS inclusion in the LV onboard equipment on the tactical and technical characteristics, reliability, operational properties and LV functioning;

d) determination of the ballistic capabilities of the ABDS for the displacement implementation of the coordinates of the WS point of fall by the value $\Delta\vec{R}_i = \vec{R}_{opt}(x, y) - \vec{R}_{pr}(x_i, y_i)$.

If the first two items a), b) are realizable within the existing IAS_{lv}, then the implementation of positions c), d) will require certain costs and time for the ABDScreation.

It is assumed that the ABDS development and its installation on Russian LV, in accordance with the proposed concept is objectively necessary, since the existing concept of design and operation of Russian LV with LPE does not satisfy a number of modern requirements. This follows from the analysis of the development of the trend of world rocket construction [1-10], in particular, the continuous increase in the requirements for environmental safety by both international and Russian legislation, increasing competition in the market of launch vehicles [1-10].

In accordance with the formulated concept of improving the ecological safety of LV with the main LPE [10, 11, 20, 22], it is proposed to develop an additional ABDS, which is assigned the main part to ensure the specified indicators for the ecological safety of LV in the WS impact area:

- extraction of unused fuel residues in tanks and WS lines after cutting off the main LPE on the WS trajectory of descending based on the technology of their transfer from the gas-liquid phase to the gas-vapor mixture [20];
- use of energy resources in the recovered vapor-gas mixture from fuel tanks to solve the problem of controlled descent of the WS [17, 18];
- development of algorithms for controlling the gas-reactive system, ensuring the WS descent at the specified point in the impact area from the condition of minimum costs for compensation of environmental damage $\{ C_i^{sa}[\vec{R}_i(x_i, y_i)] \}$.

4 Interaction of IAS_{cd} and IAS_{lv}

The interaction between IAS_{cd} and IAS_{lv}, like any information exchange between complex technical systems, has an iterative character, which can be divided into several stages and levels, both with the readiness of each of the IAS and the current tasks to be solved by each IAS.

1. At the current level, the primary task is to create an IAS_{ia} and to create a database for each impact area of the most acceptable WS points of fall from the condition min min$\{ C_i^{sa}[\vec{R}_i(x_i, y_i)]$.

2. Stages of interaction IAS will be determined by the creation terms of both mathematical models, software products, and material systems that implement them, in particular, the IAS_{ia} database, the degree of readiness of the ABDS.

3. The received information is necessary for conducting research within the framework of the IAS_{lv},IAS_{lv}^{es} for the following purposes:

a) the synthesis of various programs for the LV movement control in the launching phase, without taking into account the limitations on the WS impact areas (calculation $\vec{R}_{opt}(x, y)$);

b) an estimate of the distance $\Delta\vec{R}$ between $\vec{R}_{opt}(x, y)$ and the recommended WS points of fall, obtained in the IAS_{ia}, from the condition min $\{ C_i^{sa}[\vec{R}_i(x_i, y_i)] \}$;

c) development of proposals for changing the design and construction parameters of the WS for the maneuver implementation on the trajectory of descent.

4. The hierarchy of each IAS, the levels of interaction of the IAS_{ia} and IAS_{lv}

In Fig. 3 shows a general schematic diagram of information flows between the IAS_{ia} and IAS_{lv}

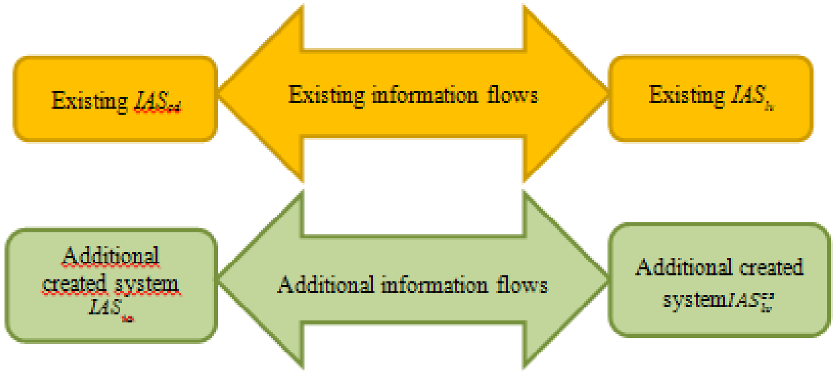

Figure 3 - The general schematic diagram of information flows between IAS_{cd}, IAS_{lv}, IAS_{ia},IAS_{lv}^{es}

Realization of the presented concept of increasing the ecological safety of the launch vehicle with the main LPE will significantly reduce the ecological load on the environment in the impact areas of the Baikonur cosmodrome due to a drastic reduction in the areas of impact areas (controlled descent of the WS), a significant reduction in the probability of vegetation fires (due to the almost complete recovery of liquid residues fuel), the choice of the safest (from the ecological point of view) points of the WS fall on the territory of the designated impact area. The volumes and costs of the IAS_{ia},IAS_{lv}^{es}creation will be determined at the next stages of the research.

5 Conclusions

1 The analysis of modern tendencies of increase of ecological safety of LV with LPE is carried out. The main factors affecting the level of environmental damage in the impact area of of the WS are given.

2 The concept of reducing the technogenic impact in the impact areas of the Baikonur Cosmodrome for non-reusable non-escaped WS is formulated, based on the operational recommendations of the IAS_{ia} to the IAS_{lv}, the composition of the information for exchange between the IAS_{ia} and IAS_{lv} is determined.

3 Proposals have been developed for the development of the design methodology for IAS_{ia} to assess the technogenic impact of LV launching on the selected fall area integrated into the general information analytic system of the Baikonur cosmodrome.

4 Proposals for theIAS_{lv} creation to improve the ecological safety of LV with main LPE in the impact areas are developed on the basis of upgrading the control programs for LV launches at the active phase of the launch trajectory, the programs for controlling the movement of the WS at the atmospheric portion of the descent trajectory, and the use of ABDS.

6 Gratitude

The research was carried out with the support of the grant of the Ministry of Education and Science of the Republic of Kazakhstan AP05131162 of 02.02.2018 and the state contract of the state corporation Roskosmos No.4770233880271660000450 of 28.08.16.

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РАЗРАБОТКА БИЗНЕС-ПРОЦЕССА ИНФОРМАЦИОННО-АНАЛИТИЧЕСКИХ СИСТЕМ КОСМОДРОМА БАЙКОНУР И ПРОЕКТИРОВАНИЯ РАКЕТЫ-НОСИТЕЛЯ ДЛЯ ПОВЫШЕНИЯ ЭКОЛОГИЧЕСКОЙ БЕЗОПАСНОСТИ В РАЙОНАХ ПАДЕНИЯ ОТРАБОТАВШИХ СТУПЕНЕЙ

Аннотация. Проведён анализ существующих информационно-аналитических системы (ИАС) космодрома Байконур ИАСкд и проектирования ракет-носителей (РН) ИАСрн. Показаны основные источники возникновения техногенного воздействия пусков РН с маршевыми ЖРД в районах падения отработавших ступеней (ОС). Предложена концепция модернизации существующих ИАСкд и ИАСрн, обеспечивающая снижение техногенного воздействия для одноразовых неспасаемых ОС, основанная на оперативных рекомендациях создаваемых ИАСрп и по обеспечению пожаровзрывобезопасности ОС, снижения размеров площади падения ОС, возможности манёвра ОС для изменения района падения.Разработаны предложения по модернизации существующей ИАСкд и концепции проектирования одноразовых РН, исходя из условий повышения экологической безопасности.

Ключевые слова:техногенное воздействие, районы падения, информационная аналитическая система, отработавшая ступень ракеты-носителя, компоненты ракетного топлива

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«БАЙҚОҢЫР» ҒАРЫШ АЙЛАҒЫНЫҢ АҚПАРАТТЫҚ-ТАЛДАУ ЖҮЙЕЛЕРІ ҮШІН БИЗНЕС-ҮДЕРІСТІ ДАМЫТУ ЖӘНЕ ҚҰЛАУ АЙМАҚТАРДА ӨТЕЛГЕН САТЫЛАРДЫҢ ЭКОЛОГИЯЛЫҚ ҚАУІПСІЗДІКТІ ЖАҚСАРТУ ҮШІН ЗЫМЫРАН ТАСЫМАЛДАУШЫЛАРДЫ ЖОБАЛАУ

Аннотация. Байқоңыр ғарыш айлағының қазіргі заманғы ақпараттық-талдау жүйелерін (АТЖ) және АТЖ$_{\text{га}}$ ұшыру аппараттарын жобалауды талдау жүргізілді. Зымыран қозғалтқыштары бар ұшыру аппараттарының өтелген сатылардағы (ӨТ) антропогендік әсерінің негізгі көздері көрсетілген. АТЖ-мен жасалған жедел ұсыныстарды негізге ала отырып, ОС-ның өрт және жарылыс қауіпсіздігін қамтамасыз ету, ОҚ-ның құлау аймағының көлемін азайту, құлау аймағын өзгерту үшін ӨС-ты маневр жасау, бір реттік қауіпсіздіктегі ӨС-ға антропогендік әсерін төмендетуді қамтамасыз ететін қолданыстағы АТЖ$_{\text{га}}$ және АТЖ$_{\text{зт}}$ жаңғырту тұжырымдамасы. Қолданыстағы АТЖ$_{\text{га}}$ жаңғырту және экологиялық қауіпсіздікті жетілдіруге негізделген бір реттік бірліктерді жобалау тұжырымдамасы бойынша ұсыныстар әзірленді.

Түйін сөздер: технологендік әсерлер, құлау аймағы, ақпараттық-талдау жүйесі, зымыран тасымалдаушының өтелген сатысы, зымырандық отын компоненттері

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N E W S
OF THE NATIONAL ACADEMY OF SCIENCES OF THE REPUBLIC OF KAZAKHSTAN
PHYSICO-MATHEMATICAL SERIES
ISSN 1991-346X **https://doi.org/10.32014/2019.2518-1726.2**
Volume 1, Number 323 (2019), 14 – 21

UDK 517.951
MRNTI 27.31.15

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ON THE INITIAL-BOUNDARY VALUE PROBLEM FOR SYSTEM OF THE PARTIAL DIFFERENTIAL EQUATIONS OF FOURTH ORDER

Abstract. A initial-boundary value problem for system of the partial differential equations of fourth order is considered. We study the existence of classical solutions to the initial-boundary value problem for system of the partial differential equations of fourth order and offer the methods for finding its approximate solutions. Sufficient conditions for the existence and uniqueness of a classical solution to the initial-boundary value problem for system of the partial differential equations of fourth order are set. By introducing of a new unknown functions, we reduce the considered problem to an equivalent problem consisting of a nonlocal problem for the system of hyperbolic equations of second order with functional parameters and the integral relations. We offer the algorithm for finding an approximate solution to the investigated problem and prove its convergence. Sufficient conditions for the existence of unique solution to the equivalent problem with parameters are established. Conditions of unique solvability to the initial-boundary value problem for system of the partial differential equations of fourth order are obtained in the terms of initial data. Separately, the result is given for the initial-periodic in time boundary value problem.

Keywords: system of the partial differential equations of fourth order, initial-boundary value problem, nonlocal problem, system of the hyperbolic equations of second order, solvability, algorithm.

1. Introduction. Currently, the problems of mathematical physics connected with the description of wave motion of liquids of different nature are drawn by great attention. This interest is caused not only by big applied importance of these problems, but their new theoretical and mathematical content often do not have analogues in the classical mathematical physics. One of the important classes of such problems are the initial-boundary value problems for fourth order partial differential equations. To date, various methods for researching and solving the initial-boundary value problems for fourth order partial differential equations of hyperbolic and composite types are developed in [1-12]. In order to investigate various boundary value problems for fourth order partial differential equations along with the classical methods of mathematical physics (the Fourier method, the method of Green's functions, Poincare's metric concept) we apply the method of differential inequalities and other methods of qualitative theory of ordinary differential equations. Based on them, the conditions for solvability of considered boundary value problems are obtained, and the ways for finding their solutions are offered. Fourth order system of partial differential equations began to be studied relatively recently.

In the present work we consider system of the partial differential equations of fourth order at the rectangular domain. Boundary condition for time variable are specified as a combination of values from the partial derivatives of required solution on third orders by spatial variable. We investigate the questions of existence and uniqueness of the classical solution to initial-boundary value problem for system of the partial differential equations of fourth order and its applications.

2. Methods. For solve to considered problem we use a method of introduction additional functional parameters [13-29]. The original problem is reduced to an equivalent problem consisting of nonlocal problem for system of the hyperbolic equations of second order with functional parameters and integral relations. Sufficient conditions for the unique solvability to investigated problem are established in the terms of initial data. Algorithms for finding solution to the equivalent problem are constructed. Conditions of unique solvability to initial-boundary value problem for system of partial differential equations of fourth order are established in the terms of system's coefficients and boundary matrices. Separately, the result is given for the initial-periodic in time boundary value problem.

Note that, in [30, 31] a similar approach has been applied to the initial-boundary value problem and nonlocal problem for the system of partial differential equations of third order.

2. Statement of problem. At the domain $\Omega=[0,T]\times[0,\omega]$ we consider the initial-periodic boundary value problem for system of the partial differential equations of fourth order in the following form

$$\frac{\partial^4 u}{\partial t\partial x^3}=A_1(t,x)\frac{\partial^3 u}{\partial x^3}+A_2(t,x)\frac{\partial^3 u}{\partial t\partial x^2}+A_3(t,x)\frac{\partial^2 u}{\partial x^2}+A_4(t,x)\frac{\partial^2 u}{\partial t\partial x}+A_5(t,x)\frac{\partial u}{\partial x}+$$
$$+A_6(t,x)\frac{\partial u}{\partial t}+A_7(t,x)u+f(t,x),\qquad (t,x)\in\Omega, \tag{1}$$

$$\frac{\partial^3 u(0,x)}{\partial x^3}=K(x)\frac{\partial^3 u(T,x)}{\partial x^3}+\varphi(x),\qquad x\in[0,\omega], \tag{2}$$

$$u(t,0)=\psi_0(t),\qquad t\in[0,T], \tag{3}$$

$$\frac{\partial u(t,x)}{\partial x}\Big|_{x=0}=\psi_1(t),\qquad t\in[0,T], \tag{4}$$

$$\frac{\partial^2 u(t,x)}{\partial x^2}\Big|_{x=0}=\psi_2(t),\qquad t\in[0,T], \tag{5}$$

where $u(t,x)=col(u_1(t,x),u_2(t,x),...,u_n(t,x))$ is unknown function, the $n\times n$-matrices $A_i(t,x)$, $i=\overline{1,7}$, and n-vector function $f(t,x)$ are continuous on Ω, the $n\times n$-matrix $K(x)$ and n-vector-function $\varphi(x)$ are continuous on $[0,\omega]$, the n-vector-functions $\psi_0(t)$, $\psi_1(t)$ and $\psi_2(t)$ are continuously differentiable on $[0,T]$. The initial data satisfy the condition of approval.

A function $u(t,x)\in C(\Omega,R^n)$ having partial derivatives $\frac{\partial u(t,x)}{\partial x}\in C(\Omega,R^n)$, $\frac{\partial u(t,x)}{\partial t}\in C(\Omega,R^n)$, $\frac{\partial^2 u(t,x)}{\partial x^2}\in C(\Omega,R^n)$, $\frac{\partial^2 u(t,x)}{\partial t\partial x}\in C(\Omega,R^n)$, $\frac{\partial^3 u(t,x)}{\partial x^3}\in C(\Omega,R^n)$, $\frac{\partial^3 u(t,x)}{\partial t\partial x^2}\in C(\Omega,R^n)$, $\frac{\partial^4 u(t,x)}{\partial t\partial x^3}\in C(\Omega,R^n)$, is called a classical solution to problem (1)--(5) if it satisfies system (1) for all $(t,x)\in\Omega$, and boundary conditions (2)--(5).

We will investigate the questions of existence and uniqueness of the classical solutions to the initial-boundary value problem for system of the partial differential equations of fourth order (1)--(5) and the approaches of constructing its approximate solutions. For this goals, we applied the method of introduction additional functional parameters proposed in [13-31] for solving of various nonlocal problems for systems of hyperbolic equations with mixed derivatives. Considered problem is provided to nonlocal problem for the system of hyperbolic equations of second order including additional functions and integral relation. The algorithm for finding the approximate solution of the investigated problem is proposed and its convergence proved. Sufficient conditions of the existence unique classical solution to problem (1)--(5) are obtained in the terms of initial data.

3. Scheme of the method and reduction to equivalent problem. We introduce a new unknown functions $v(t,x)=\dfrac{\partial u(t,x)}{\partial x}$, $w(t,x)=\dfrac{\partial^2 u(t,x)}{\partial x^2}$ and rewrite the problem (1)--(5) in the following form

$$\frac{\partial^2 w}{\partial t\partial x}=A_1(t,x)\frac{\partial w}{\partial x}+A_2(t,x)\frac{\partial w}{\partial t}+A_3(t,x)w+f(t,x)+$$

$$+A_4(t,x)\frac{\partial v(t,x)}{\partial t}+A_5(t,x)v(t,x)+A_6(t,x)\frac{\partial u(t,x)}{\partial t}+A_7(t,x)u(t,x),\qquad (t,x)\in\Omega,\qquad (6)$$

$$\frac{\partial w(0,x)}{\partial x}=K(x)\frac{\partial w(T,x)}{\partial x}+\varphi(x),\qquad x\in[0,\omega],\qquad (7)$$

$$w(t,0)=\psi_2(t),\qquad t\in[0,T],\qquad (8)$$

$$v(t,x)=\psi_0(t)+\int_0^x w(t,\xi)d\xi,\quad u(t,x)=\psi_0(t)+\psi_1(t)x+\int_0^x\int_0^\xi w(t,\xi_1)d\xi_1 d\xi,\quad (t,x)\in\Omega.\qquad (9)$$

Here the conditions (3) and (4) are taken into account in (9).

A triple functions $(w(t,x),v(t,x),u(t,x))$, where the function $w(t,x)\in C(\Omega,R^n)$ has partial derivatives $\dfrac{\partial w(t,x)}{\partial x}\in C(\Omega,R^n)$, $\dfrac{\partial w(t,x)}{\partial t}\in C(\Omega,R^n)$, $\dfrac{\partial^2 w(t,x)}{\partial t\partial x}\in C(\Omega,R^n)$, the functions $v(t,x)\in C(\Omega,R^n)$ and $u(t,x)\in C(\Omega,R^n)$ have partial derivatives $\dfrac{\partial v(t,x)}{\partial x}\in C(\Omega,R^n)$, $\dfrac{\partial v(t,x)}{\partial t}\in C(\Omega,R^n)$, $\dfrac{\partial^2 v(t,x)}{\partial x^2}\in C(\Omega,R^n)$, $\dfrac{\partial^2 v(t,x)}{\partial t\partial x}\in C(\Omega,R^n)$, $\dfrac{\partial^3 v(t,x)}{\partial t\partial x^2}\in C(\Omega,R^n)$, $\dfrac{\partial u(t,x)}{\partial x}\in C(\Omega,R^n)$, $\dfrac{\partial u(t,x)}{\partial t}\in C(\Omega,R^n)$, $\dfrac{\partial^2 u(t,x)}{\partial x^2}\in C(\Omega,R^n)$, $\dfrac{\partial^2 u(t,x)}{\partial t\partial x}\in C(\Omega,R^n)$, $\dfrac{\partial^3 u(t,x)}{\partial t\partial x^2}\in C(\Omega,R^n)$, $\dfrac{\partial^3 u(t,x)}{\partial x^3}\in C(\Omega,R^n)$, $\dfrac{\partial^4 u(t,x)}{\partial t\partial x^3}\in C(\Omega,R^n)$, is called a solution to problem (6)--(9) if it satisfies of the system of hyperbolic equations of second order (6) for all $(t,x)\in\Omega$, the boundary conditions (7), (8), and the integral relations (9).

At fixed $v(t,x)$ and $u(t,x)$ the problem (6)--(8) is a nonlocal problem for the system of hyperbolic equations with respect to $w(t,x)$ on Ω. The integral relations (9) allow us to determine the unknown functions $v(t,x)$ and $u(t,x)$ for all $(t,x)\in\Omega$.

4. Algorithm. The unknown function $w(t,x)$ will be determined from nonlocal problem for the system of hyperbolic equations (6)--(8). The unknown functions $v(t,x)$ and $u(t,x)$ will be found from integral relations (9).

If we know the functions $v(t,x)$ and $u(t,x)$, then from nonlocal problem (6)--(8) find the function $w(t,x)$. Conversely, if we known the functions $v(t,x)$ and $u(t,x)$, then from nonlocal problem (6)--(8) we find the function $w(t,x)$. Since the functions $v(t,x)$, $u(t,x)$ and $v(t,x)$ are unknowns together, for finding of the solution to problem (6)--(9) we use an iterative method. The solution to problem (6)--(9) is the triple functions $(w^*(t,x),v^*(t,x),u^*(t,x))$ we defined as a limit of sequence of triples $(w^{(k)}(t,x),v^{(k)}(t,x),u^{(k)}(t,x))$, $k=0,1,2,...$, according to the following algorithm:

Step 0. 1) Suppose in the right-hand part of the system (6) $\frac{\partial v(t,x)}{\partial t} = \dot{\psi}_0(t)$, $v(t,x) = \psi_0(t)$, $\frac{\partial u(t,x)}{\partial t} = \dot{\psi}_0(t) + \dot{\psi}_1(t)x$, and $u(t,x) = \psi_0(t) + \psi_1(t)x$, from nonlocal problem (6)--(8) we find the initial approximation $w^{(0)}(t,x)$ for all $(t,x) \in \Omega$;

2) From the integral relations (9) under $w(t,x) = w^{(0)}(t,x)$, we find the functions $v^{(0)}(t,x)$ and $u^{(0)}(t,x)$ for all $(t,x) \in \Omega$.

Step 1. 1) Suppose in the right-hand part of system (6) $\frac{\partial v(t,x)}{\partial t} = \frac{\partial v^{(0)}(t,x)}{\partial t}$, $v(t,x) = v^{(0)}(t,x)$, $\frac{\partial u(t,x)}{\partial t} = \frac{\partial u^{(0)}(t,x)}{\partial t}$, and $u(t,x) = u^{(0)}(t,x)$, from nonlocal problem (6)--(8) we find the first approximation $w^{(1)}(t,x)$ for all $(t,x) \in \Omega$.

2) From the integral relations (9) under $w(t,x) = w^{(1)}(t,x)$, we find the functions $v^{(1)}(t,x)$ and $u^{(1)}(t,x)$ for all $(t,x) \in \Omega$.

And so on.

Step k. 1) Suppose in the right-hand part of system (6) $\frac{\partial v(t,x)}{\partial t} = \frac{\partial v^{(k-1)}(t,x)}{\partial t}$, $v(t,x) = v^{(k-1)}(t,x)$, $\frac{\partial u(t,x)}{\partial t} = \frac{\partial u^{(k-1)}(t,x)}{\partial t}$, and $u(t,x) = u^{(k-1)}(t,x)$, from nonlocal problem (6)--(8) we find the k-th approximation $w^{(k)}(t,x)$ for all $(t,x) \in \Omega$:

$$\frac{\partial^2 w^{(k)}}{\partial t \partial x} = A_1(t,x)\frac{\partial w^{(k)}}{\partial x} + A_2(t,x)\frac{\partial w^{(k)}}{\partial t} + A_3(t,x)w^{(k)} + f(t,x) +$$

$$+ A_4(t,x)\frac{\partial v^{(k-1)}(t,x)}{\partial t} + A_5(t,x)v^{(k-1)}(t,x) + A_6(t,x)\frac{\partial u^{(k-1)}(t,x)}{\partial t} + A_7(t,x)u^{(k-1)}(t,x), \quad (t,x) \in \Omega, \quad (10)$$

$$\frac{\partial w^{(k)}(0,x)}{\partial x} = K(x)\frac{\partial w^{(k)}(T,x)}{\partial x} + \varphi(x), \qquad x \in [0,\omega], \quad (11)$$

$$w^{(k)}(t,0) = \psi_2(t), \qquad t \in [0,T]. \quad (12)$$

2) From the integral relations (9) under $w(t,x) = w^{(k)}(t,x)$, we find the functions $v^{(k)}(t,x)$ and $u^{(k)}(t,x)$ for all $(t,x) \in \Omega$:

$$v^{(k)}(t,x) = \psi_0(t) + \int_0^x w^{(k)}(t,\xi)d\xi, \quad u^{(k)}(t,x) = \psi_0(t) + \psi_1(t)x + \int_0^x\int_0^\xi w^{(k)}(t,\xi_1)d\xi_1 d\xi, \quad (t,x) \in \Omega. \quad (13)$$

Here $k = 1,2,3,...$.

5. The main results. The following theorem gives conditions of feasibility and convergence of the constructed algorithm and the conditions of the existence unique solution to problem (6)--(9).

Theorem 1. *Suppose that*

i) the $n \times n$*-matrices* $A_i(t,x)$, $i = \overline{1,7}$, *and* n*-vector function* $f(t,x)$ *are continuous on* Ω;

ii) the $n \times n$*-matrix* $K(x)$ *and* n*-vector-function* $\varphi(x)$ *are continuous on* $[0,\omega]$;

iii) the n*-vector-functions* $\psi_0(t)$, $\psi_1(t)$ *and* $\psi_2(t)$ *are continuously differentiable on* $[0,T]$;

iv) the $n \times n$*-matrix* $Q(x) = I - K(x)\left[I + \int_0^T A_1(\tau,x)d\tau\right]$ *is invertible for all* $x \in [0,\omega]$, *where* I *is unit matrix on dimension* n.

Then the nonlocal problem for system of the hyperbolic equations with parameters (6)--(9) has a unique solution $(w^*(t,x), v^*(t,x), u^*(t,x))$ *as a limit of sequences* $(w^{(k)}(t,x), v^{(k)}(t,x), u^{(k)}(t,x))$ *defining by the algorithm proposed above for* $k = 0,1,2,...$.

Proof. Let the conditions *i) - iv)* of the Theorem be satisfied. From the 0th step of the above algorithm and Theorem 1 from [21] it follows that the nonlocal problem for system of the hyperbolic equations

$$\frac{\partial^2 w}{\partial t \partial x} = A_1(t,x)\frac{\partial w}{\partial x} + A_2(t,x)\frac{\partial w}{\partial t} + A_3(t,x)w + f(t,x) + A_4(t,x)\dot{\psi}_0(t) + A_5(t,x)\psi_0(t) +$$
$$+ A_6(t,x)[\dot{\psi}_0(t) + \dot{\psi}_1(t)x] + A_7(t,x)[\psi_0(t) + \psi_1(t)x], \qquad (t,x) \in \Omega, \tag{14}$$

$$\frac{\partial w(0,x)}{\partial x} = K(x)\frac{\partial w(T,x)}{\partial x} + \varphi(x), \qquad x \in [0,\omega], \tag{15}$$

$$w(t,0) = \psi_2(t), \qquad t \in [0,T] \tag{16}$$

has a unique classical solution $w^{(0)}(t,x)$ for all $(t,x) \in \Omega$.

Further we determine the functions $v^{(0)}(t,x)$ and $u^{(0)}(t,x)$ from the integral relations

$$v^{(0)}(t,x) = \psi_0(t) + \int_0^x w^{(0)}(t,\xi)d\xi, \quad u^{(0)}(t,x) = \psi_0(t) + \psi_1(t)x + \int_0^x\int_0^\xi w^{(0)}(t,\xi_1)d\xi_1 d\xi$$

for all $(t,x) \in \Omega$. Functions $v^{(0)}(t,x)$ and $u^{(0)}(t,x)$ together with their partial derivatives $\frac{\partial v^{(0)}(t,x)}{\partial t}$ and $\frac{\partial u^{(0)}(t,x)}{\partial t}$, respectively, are continuous on Ω.

Continuing the iterative process according to the above algorithm, we define successive approximations $w^{(k)}(t,x)$, $v^{(k)}(t,x)$ and $u^{(k)}(t,x)$ for all $(t,x) \in \Omega$ and $k = 1,2,...$.

The conditions *i) - iv)* of Theorem provide the uniform convergence on Ω of the sequences $\{w^{(k)}(t,x)\}$, $\{v^{(k)}(t,x)\}$ and $\{u^{(k)}(t,x)\}$ as $k \to \infty$ to functions $w^*(t,x)$, $v^*(t,x)$ and $u^*(t,x)$, respectively, for all $(t,x) \in \Omega$. In addition, there are finite limits of sequences of their partial derivatives as $k \to \infty$.

The triple founded functions $(w^*(t,x), v^*(t,x), u^*(t,x))$ has all the required continuous partial derivatives on Ω and be solution to problem (6)—(9). Uniqueness of solution to problem (6)—(9) is proved by method of contradiction.

Theorem 1 is proved.

From the equivalence of problems (6)—(9) and (1)—(5) it follows

Theorem 2. *Suppose that the conditions i) - iv) of Theorem 1 are fulfilled.*

Then the initial-periodic boundary value problem for system of the partial differential equations of fourth order (1)--(5) has a unique classical solution $u^*(t,x)$.

For $K(x) = I$ and $\varphi(x) = 0$ we obtain the initial-periodic boundary value problem for system of the partial differential equations of fourth order (1), (3)-(5) with condition

$$\frac{\partial^3 u(0,x)}{\partial x^3} = \frac{\partial^3 u(T,x)}{\partial x^3}, \qquad x \in [0,\omega]. \tag{2'}$$

Then the following assertion is true.

Theorem 3. *Suppose that*

1) the $n \times n$ *-matrices* $A_i(t,x)$, $i = \overline{1,7}$, *and* n *-vector function* $f(t,x)$ *are continuous on* Ω;

2) the n *-vector-functions* $\psi_0(t)$, $\psi_1(t)$ *and* $\psi_2(t)$ *are continuously differentiable on* $[0,T]$;

3) *the* $n \times n$ *-matrix* $Q(x) = \int_0^T A_1(\tau,x)d\tau$ *is invertible for all* $x \in [0,\omega]$.

Then the initial-periodic boundary value problem for system of the partial differential equations of fourth order (1), (2'), (3)--(5) has a unique classical solution.

Funding. This results are partially supported by grant of the Ministry education and science of Republic Kazakhstan No. AP 05131220 for 2018-2020 years.

УДК 517.951
МРНТИ 27.31.15

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О НАЧАЛЬНО-КРАЕВОЙ ЗАДАЧЕ ДЛЯ СИСТЕМЫ ДИФФЕРЕНЦИАЛЬНЫХ УРАВНЕНИЙ В ЧАСТНЫХ ПРОИЗВОДНЫХ ЧЕТВЕРТОГО ПОРЯДКА

Аннотация. Рассматривается начально-краевая задача для системы дифференциальных уравнений в частных производных четвертого порядка. Исследуются вопросы существования классического решения начально-краевой задачи для системы дифференциальных уравнений в частных производных четвертого порядка и предлагаются методы нахождения их приближенных решений. Установлены достаточные условия существования и единственности классического решения начально-краевой задачи для системы дифференциальных уравнений в частных производных четвертого порядка. Путем введения новых неизвестных функций исследуемая задача сведена к эквивалентной задаче, состоящей из нелокальной задачи для системы гиперболических уравнений второго порядка с функциональными параметрами и интегральных соотношений. Предложены алгоритмы нахождения приближенного решения исследуемой задачи и доказана их сходимость. Установлены достаточные условия существования единственного решения эквивалентной задачи с параметрами. Условия однозначной разрешимости начально-краевой задачи для системы дифференциальных уравнений в частных производных четвертого порядка получены в терминах исходных данных. Отдельно приводится результат для начально-периодической по времени краевой задачи.

Ключевые слова: система дифференциальных уравнений в частных производных четвертого порядка, начально-краевая задача, нелокальная задача, система гиперболических уравнений второго порядка, разрешимость, алгоритм.

УДК 517.951
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ТӨРТІНШІ РЕТТІ ДЕРБЕС ТУЫНДЫЛЫ ДИФФЕРЕНЦИАЛДЫҚ ТЕҢДЕУЛЕР ЖҮЙЕСІ ҮШІН БАСТАПҚЫ - ШЕТТІК ЕСЕП ТУРАЛЫ

Аннотация. Төртінші ретті дербес туындылы дифференциалдық теңдеулер жүйесі үшін бастапқы-шеттік есеп қарастырылады. Төртінші ретті дербес туындылы дифференциалдық теңдеулер жүйесі үшін бастапқы-шеттік есептің классикалық шешімінің бар болуы мәселелері мен олардың жуық шешімдерін табу әдістері зерттелген. Төртінші ретті дербес туындылы дифференциалдық теңдеулер жүйесі үшін бастапқы-шеттік есептің классикалық шешімінің бар болуы мен жалғыздығының жеткілікті шарттары тағайындалған. Жаңа белгісіз функциялар енгізу жолымен зерттеліп отырған есеп гиперболалық теңдеулер жүйесі үшін параметрлері бар бейлокал есептен және интегралдық қатынастардан тұратын пара-пар есепке келтірілген.

Зерттеліп отырған есептің жуық шешімін табу алгоритмдері ұсынылған және олардың жинақтылығы дәлелденген. Параметрлері бар пара-пар есептің жалғыз шешімінің бар болуының жеткілікті шарттары тағайындалған. Төртінші ретті дербес туындылы дифференциалдық теңдеулер жүйесі үшін бастапқы-шеттік есептің бірмәнді шешілімділігінің шарттары бастапқы берілімдер терминінде алынған. Бастапқы-уақыт бойынша периодты шеттік есеп үшін нәтиже жеке келтірілген.

Түйін сөздер: Төртінші ретті дербес туындылы дифференциалдық теңдеулер жүйесі, бастапқы-шеттік есеп, бейлокал есеп, екінші ретті гиперболалық теңдеулер жүйесі, шешілімділік, алгоритм.

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N E W S
OF THE NATIONAL ACADEMY OF SCIENCES OF THE REPUBLIC OF KAZAKHSTAN
PHYSICO-MATHEMATICAL SERIES
ISSN 1991-346X **https://doi.org/10.32014/2019.2518-1726.3**
Volume 1, Number 323 (2019), 22 – 27

UDC 372.85

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THE POSSIBILITY OF CREATING LEARNING SITUATIONS AND LEARNING TASKS IN LEARNING MATHEMATICS AT SCHOOL

Abstract. The article outlines the problem of methodological tools to assist the growing human mastery of them a frame of reference of knowledge. It is shown that one of these landmarks when learning mathematical activity can become a so-called "hidden", or rather "transformed" form, the original sense and meaning of which were lost for the student. For their awareness of students benefits such methodological tool, as a learning situation analysis pupils which leads to the generation of learning tasks. Target the intent of the article is an invitation to collaborate in the creation of a set of learning situations and tasks for the development of the students. The main purpose of this collection is to contribute to the training and education of mathematics

Key words:educational mathematical activity, functional literacy activities, learning situation, learning task,methodical tools.

Abstract. The article outlines the problem of methodological tools to assist the growing man in mastering them a system of reference points. It is shown that one of these guidelines in the mastery of educational mathematical activity can be the so-called "hidden", or rather "transformed" forms, the original meaning and meaning of which were lost for the student. For their awareness of students benefits from such a methodological tool as the learning situation, the analysis of which with students leads to a series of learning tasks. The purpose of the article is an invitation to cooperation in the creation of a set of educational situations and tasks for the development of students. The main purpose of this collection is to promote learning and education in mathematics.

Key words: educational math activities, functional literacy activities, learning situation, learning task, methodological tools

The name of the project requires a little explanation…

I remember the popular in the 80-ies of the now last century humorous scene performed by Gennady Khazanov – a funny case with a student of the culinary school in the exam:

- (Examiner.) Determine what is missing in Your borsch.

- H-h-l-e-e-BA.…

Laughter in the hall ... What and why did people laugh so contagiously?

One version: over the image of a careless student, skillfully represented by a talented artist: sometimes-such a student!! And do nothing…

Version two: as it is taught in this College... we Have not, we are better: even the careless will answer correctly such a simple question…

You can find other explanations. But they all come down to one thing: training has not reached the planned result. This-whether we want to admit it or not – has happened before, and now it occurs quite often. What are the reasons? The authors of the article adhere to another version. Its essence is that in the vast majority of cases, training (in school or University) is perceived by the student (and teaching) as imposed from somewhere above. The teacher (teacher) most often takes this for granted and explains (for himself and others): it is defined by the Program, in the end – the Ministry, it can only be changed within the specified framework and adhering to the established techniques... so required!!

And student? Most often he hears the phrase: "let's Solve the problem!"

Dear reader, at least for a moment imagine yourself as a student and first whisper, then louder and "in a stretch" say: for-da-cha. We assume that carefully, several times in a row uttered and sincerely and carefully tracked this word in many of you (except, perhaps, professionals from mathematics) will cause, if not unpleasant, then at least, a sense of obsession. And situation?

In a number of works of one of authors of this article, including [3,14,14,15,16], it is proved that the educational situation (MOUSTACHE) on the pedagogical role represents educational analog of so-called vital for the subject of a situation. The latter, in turn, is always a situation of choice, containing some difficulty, overcoming or not overcoming which leads to the formation of his worldview (as a holistic quality of personality – unity of emotion-attitude, representation-knowledge, "program" of actions in their relationships) micromechanism of activity of resolving difficulties. This is often manifested in the form of motivations to knowledge, attitudes, positions in relation to something, someone and-most importantly-activities to resolve the situation. And when the subject is involved in the activity begin to "work" all its components: there is a motive, aware of the goal, outlined the means and actions to achieve it, formulated a number of tasks as a plan for further work.

Thus, our position is that learning objectives (KM) originate within the framework of the HS or give rise to it as its organizing core. This was the motive and the basis for the creation of the project: a set of KM and KM.

In the educational process it is advisable to specifically create and purposefully use educational situations (US) as a pedagogical tool to assist cognitive activity and the emerging system of reference points of knowledge of a growing person.

According to [5], one of the most important results of teaching mathematics and mathematics education should be the ability of students to learn about the world and themselves in it [3,4,7,19,20]. But it requires support, first, of previously acquired experience of cognitive activity, secondly, on the achievements of such experience, the components of which, of course, are mastered earlier by man and proven benchmarks such activities and their results in the form of actions to achieve result of acquired knowledge, methods and means of knowledge. The set of such guidelines and actions assigned to a particular person, we call it functional literacy. It follows that the most important result of education, especially at the initial stage of training, should be the mastery of functional literacy of cognitive activity. But in terms of training it is possible only when the student resolves analogues of vital situations for him, that is, educational situations (S) [3, 4]. Therefore, in terms of teaching any academic disciplines, it is important not only to consider the possibility of the emergence of the MOUSTACHE, but need a special work of the teacher to create them and on their basis, preferably together with students, to identify within them and the formulation on this basis of a variety of TIES, primarily as "tasks for themselves". It is assumed that the created methodical project will help the teacher (especially the beginner) in the organization of cognitive activity of students or students in the study with them of certain fragments of educational mathematical disciplines. In this regard, our further task is to acquaint the reader with the examples of SS created by us or "peeped" in the experience of the best teachers and, as an intermediate result, to set possible grounds for their development.

Case study (CS) is created in the interaction of teacher (St), student (UK) and linking them to some of the works of culture (PC). Note that these designations are quite suitable in terms of both school and University education.

PC contains (or should contain) in itself "fragile confrontation", "the darkness as absence of light, painfully we razbiralsya" [5, p. 198] and acts like a materialized media (methodological tool) create the appropriate BONDS. So the structure of the SS can be represented as: SS = Ul; PC; UK □ ≈ ≈ Ul; UZ; UK□. At the same time, the UZ will be understood as the unity of two components: a certain array of content (subject) data and a set of tasks for students, coordinated with the PC and carrying in themselves and setting any functions-educational, developmental or educational. In connection with this, the types of KM are consistent with the types of KM and based on them educational and educational "super-tasks".

In accordance with the accepted understanding of the MOUSTACHE, among the educational tasks of its core (UZ) should be available such that direct the student to develop in themselves or contribute to the formation of his or her various orientations of activity and personal qualities. The latter may include those given, for example, in [4, CC. 61, 113, 157], or those which the teacher or teacher considers it necessary to form in schoolchildren or students. We will explain what was said in this regard on the examples of some educational situations, conditionally correlated with the period of training.

Situation 1 (grades 5-6). Mom for the preservation of mushrooms needed 8% solution of acetic acid. There is 70% acetic acid in the household. What advice to give mom to dilute the concentrated acid solution to the desired 8%? Is it possible to make General recommendations on how to lower (increase) the percentage of the solution of a given substance? Can you and how do you do it using math?

Situation 2 (grades 5-9). In the nearest fishery was built a pond for breeding mirror carp. Launched a fry, and after a while before the workers of the economy faced the question: what income should be expected from the sale of fish to the population? How to assign work for catching fish? How to help in solving these issues, will we be able to give reasonable recommendations, how to do it with the help of our knowledge?

Situation 3 (5-10 classes). Imagine that we are together-the inhabitants of Ancient Egypt, and we have land in the floodplain of the Nile river. Me and one of you (who is willing?!?) we have plots with a common boundary along the AN line, and together our plots have the shape of an ABCD quadrilateral. Our sites before the flooding of the river had the same area. Suppose that before the flood of the river so well managed to secure the poles in all the peaks that after the decline of the water they were found. And only the pole N disappeared without a trace. Is it possible to restore the border to AN AREA so that our plots are still the same size?

Scenario 4 (5– 9 classes). (We will help the head teacher or The choice of an effective method of counting options). Let the head teacher addressed us and asked to help in drawing up a schedule of after-hours work. It is known that in our Lyceum after classes fifth graders are engaged in three circles:

theater, natural science, dance. There is a situation-a problem: how many ways it is possible to make the schedule of extracurricular occupations?

Ul: Remember the problem we solved when calculating the three-digit number. How did we find options? What mathematical problem was solved, and what method (method) was used?

Ul: Come up with a few more similar situations…

Situation 5 (5-9 classes). 1. For four new students need to create passwords to enter the electronic journal, using the numbers 4,5,9. Is it possible? What technique is better to use? Formulate mathematical problems that you will solve at the same time?

2. In the school dining room at first cooked soup and hodgepodge, the second – cutlets, casserole and fish, the third – tea and juice. How many menu options can you advise to make the head of the dining room?

Offer your options and make the necessary math problems for your friends, classmates. Solve these problems. Explain what technique and how best to use. What knowledge of mathematics did you need?

Note that the examples given are situations, not mathematical problems – the problem still "see", to formulate, and this situation is really with the "student" person, that is addressed directly to the student. As a rule, even "weak" students (according to the teachers ' observations) "suddenly" begin to feel like participants (!) this kind of situations and begin to offer their versions of their resolution. This is the beginning of their movement to comprehend functional literacy (FGUMD) of their own educational activities, to understand their actions and the means used (including such as definitions, theorems), etc., that is – to comprehend the basics of scientific knowledge.

Concretized tasks of such types of SS and UZ are made according to the following scheme: (1) the system of the interconnected qualities which formation plans to form on a series of occupations or throughout all course of training is defined (they, as a rule, set type of SS); (2) these qualities are transferred to the form of the General questions: what needs to be made for formation of the necessary qualities? (3) Using the planned study program material, selecting the necessary array of meaningful data, taken from experience, from the media or school textbooks, and it will result in the previously mentioned series or other issues. All of this leads to a series of BONDS being singled out from this type of

MOUSTACHE as "tasks for me". Highlight next some proven experience in the types of WHISKER and respective BONDS. Next, in brackets in italics is given the main meaning of the type of MUSTACHE.

UZ type 1 (UZ-1 – reproduction of knowledge).

Given: different characteristics of real-life, occurring in nature, in the human experience, or ideal objects or phenomena (characteristics may not be related either in content or in the subject area).

TASKS: 1. Analyze the existing characteristics: name them, compare with each other; say that they mark (allocate) in these objects (phenomena). If possible, refer them to one or different groups (types, classes); describe the characteristics of words, symbols. 2. Find the mathematical relationships between some characteristics, Express the dependence found in symbolic or other form. 3. Tell and explain the relationship (connection) between the characteristics reflecting the dependence to which mathematical knowledge (arithmetic, geometry, algebra) it belongs to, tell someone or get in writing. 4. Find in textbooks or give your other examples of the dependence found.

UZ type 2 (UZ-2 – reproduction of mathematical activity).

Given: 1) a set of mathematical symbols (symbols of numbers, letters, names of figures, etc.); 2) a set of signs of mathematical actions on the corresponding mathematical objects or the relationship between them.

TASKS: 1. Using the characters from both sets, make known to you: a) formula (with the sign =); b) other expressions that do not contain signs <; =; >; d) inequality. 2. Make expressions or formulas that you have not met (violate the rules of the use of signs is prohibited!); compile using the same characters having the meaning of the statement. 3. Explain the new dependencies or statements you have received as you understand them (for example, using previously known dependencies, examples, including from life, etc.).); select a new expression or assertion. 4. If possible, find examples from familiar areas of knowledge (natural science, chemistry, etc.), from the natural world around you or come up with your own to use a new dependence, select such examples from the messages of the teacher, students or from books, including in other areas of knowledge. 5. Explain your steps that led to a new addiction or statement that you formulated. 6. Make a conclusion about the order of obtaining dependencies and found their prototypes.

ӘОЖ 372.85

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МЕКТЕПТЕ ОҚУШЫЛАРДЫ МАТЕМАТИКАҒА ОҚЫТУ БАРЫСЫНДА ОҚУ ЖАҒДАЙЛАРЫ МЕН ОҚУ МІНДЕТТЕРІН ҚҰРУ МҮМКІНДІКТЕРІ

Аннотация: Мақалада әдістемелік құралдар мәселесі бойынша өніп келе жатқан адамға танымал бағдарламалық жүйесін меңгеру белгіленеді. Көрсетілгендей, бұл осындай бағдарларды кезінде меңгеруде оқу математикалық қызметпен болуы мүмкін деп аталатын "жасырын", дәлірек айтқанда "айналым" нысандары, бастапқы мағынасы және оның мәні тап оқушы үшін жойылған. Оларды түсіну оқушыларымен пайдасы осындай әдістемелік құрал, оқу жағдайы, оны талдау, оқушылармен жеңіліс сериясына әкеледі, оқу тапсырмаларын.Мақсатты ойды – шақыру ынтымақтастық құру жинағын оқу жағдайларды және міндеттерді оқушыларды дамыту үшін. Басты мақсаты осындай жинақ – ықпал оқыту және тәрбиелеу математикамен.

Түйін сөздер: оқу математикалық қызметі, функционалдық сауаттылық қызметі, оқу жағдайы, оқу міндеті, әдістік жабдықтар.

УДК 372.85

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ВОЗМОЖНОСТИ СОЗДАНИЯ УЧЕБНЫХ СИТУАЦИЙ И УЧЕБНЫХ ЗАДАЧ В ОБУЧЕНИИ УЧАЩИХСЯ МАТЕМАТИКЕ В ШКОЛЕ

Аннотация. В статье намечается проблема методического инструментария по оказанию помощи растущему человеку в овладении им системой ориентиров познания. Показано, что одним из таких ориентиров при овладении учебной математической деятельностью могут стать так называемые «скрытые», точнее «превращённые» формы, первоначальный смысл и значение которых оказались для ученика утерянными. Для их осознания учениками приносит пользу такой методический инструмент, как учебная ситуация, анализ которой с учащимися приводит к порождению серии учебных задач. Целевой замысел статьи – приглашение к сотрудничеству в создании *комплекта* учебных ситуаций и задач для развития учащихся. Главное назначение такого сборника – способствовать **обучению и воспитанию математикой**.

Ключевые слова: учебная математическая деятельность, функциональная грамотность деятельности, учебная ситуация, учебная задача, методический инструментарий.

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N E W S
OF THE NATIONAL ACADEMY OF SCIENCES OF THE REPUBLIC OF KAZAKHSTAN
PHYSICO-MATHEMATICAL SERIES
ISSN 1991-346X **https://doi.org/10.32014/2019.2518-1726.4**
Volume 1, Number 323 (2019), 28 – 37

UDC 517.927

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NUMERICAL ANALYSIS OF THE SOLUTION OF SOME OSCILLATION PROBLEMS BY THE DECOMPOSITION METHOD

Abstract: Rectangular flat plates are one of the main elements of building structures and constructions. While solving applied problems of oscillation of rectangular flat elements then a wide class of oscillation problems occur related to various boundary-value problems: approximate oscillation equations, various boundary conditions at the edges of a flat element and initial conditions. In the theory of oscillation, an important point is to determine the frequencies of intrinsic variations, to solve problems on forced variations of a plane element, and to study the dissemination of harmonic waves in them. In this paper, we present the results on the investigation of natural and forced oscillations of flat elements taking into account the stratification of element's material of rheological viscous properties, the influence of the environment a deformable base, anisotropy, etc. The influence of these factors makes it much more difficult to study the problems of natural and forced oscillations of a flat element on dissemination of harmonic waves in them.

Key words: natural oscillations, forced oscillations, frequency equations, transcendental equations, decomposition method, relaxation time, voltage, plate.

In the study of harmonic waves in deformable bodies, there is introduced a concept of phase velocity as the rate of change of the environmental state, while the phase velocity is expressed in terms of the natural oscillation frequencies, and therefore the study of harmonic wave dissemination is directly related to the problems of determining natural shapes and frequencies of oscillation concerning flat elements.

In this paper, we present the results on the investigation of the natural and forced oscillations of flat elements taking into account the stratification of the element's material, rheological viscous properties, the influence of the environment, a deformable base, anisotropy, etc. The influence of these factors makes it much more difficult to study the problems of natural and forced oscillations of a flat element on dissemination of harmonic waves in them.

Therefore, the work is devoted to the formulation of various boundary-value problems of rectangular flat element oscillations taking into account the viscosity as well as the abovementioned factors of geometric and mechanical nature.

First of all we consider the frequency equation

$$\alpha_0 \cos(\alpha_0 l_1)\sin(\alpha_1 l_1) - \alpha_1 \sin(\alpha_0 l_1)\cos(\alpha_1 l_1) = 0. \tag{1}$$

and its equivalent equation

$$\alpha_0\alpha_1 \sum_{i=0}^{\infty}\sum_{j=0}^{\infty}(-1)^{i+j}\frac{\alpha_1^{2i}\alpha_0^{2j} - \alpha_0^{2i}\alpha_1^{2j}}{(2i+1)!(2j)!} l^{2(i+j)} = 0 \tag{2}$$

One of these frequency equations follows from the condition $a=0$ that leads to the frequency equation

$$\xi^4 - \frac{8\left[(2-\nu)\gamma + \frac{3}{2}(1-\nu)\right]}{(7-8\nu)}\xi^2 + \frac{8\gamma^2}{(7-8\nu)} = 0; \tag{3}$$

The frequency equation (3) also follows from the equation

$$\xi^4 + \frac{2}{\tau_0}\xi^3 + \frac{8}{(7-8\nu)}\left[(2-\nu)\gamma + \frac{(7-8\nu)}{8\tau_0^2} + \frac{3}{2}(1-\nu)\right]\xi^2 + \frac{12(1-\nu)}{(7-8\nu)\tau_0}\left[1+2(2-\nu)\gamma\right]\xi + \\ + \frac{8}{(7-8\nu)}\gamma^2 = 0, \tag{4}$$

for elastic plate or from equation

$$B_0\xi^4 + \frac{2B_0}{\tau_0}\xi^3 + (1 + \frac{B_0}{\tau_0^2} + B_1\gamma)\xi^2 + \frac{1}{\tau_0}(1+B_1\gamma)\xi + B_2\gamma^2 = 0 \tag{5}$$

for hinged plate.

If we consider other approximate frequency equations derived from equation (2), for example, the equation

$$\xi^2 = \frac{2\gamma + 10l^{-2}}{(2-\nu)}; \tag{6}$$

the root of which is equal to

$$\xi = \sqrt{\frac{2\gamma + 10l^{-2}}{(2-\nu)}} \tag{7}$$

The conditions of convergence (2), described by inequalities

$$\left|a_0^2 a_1^2\right| \le q_{i,j}^2 = q_{i,j}^2 = q^2\frac{(2i+3)(2j+2)}{l^2} \tag{8}$$

or

$$D^2 - E \le C_{i,j}^2 \tag{9}$$

also contain the left side of equation (3) and indicate that all the roots of the transcendental equation (2) are between the roots ξ_1 и ξ_2 and which are the lower and upper boundaries of all frequencies of the transcendental equation (1).

A similar conclusion follows from the transcendental equations

$$2 - \frac{a_0^2 + a_1^2}{a_0 a_1}\sin(a_0 l_1)\sin(a_1 l_1) - 2\cos(a_0 l_1)\cos(a_1 l_1) = 0 \tag{10}$$

Of other transcendental equations.

Thus, the natural oscillation frequencies of a rectangular hinged plate ξ_1 и ξ_2, based on an approximate equation of fourth order oscillations in derivatives are the lower and upper boundary of natural oscillation frequencies of a rectangular plate under more difficult conditions for fixing its edges.

других трансцендентных уравнений.

The obtained results belonged to the class of boundary-value problems, when two of the opposite edges of a rectangular plate are hinged, and the other two edges have different fixation conditions or are free from stresses.

If all four edges are arbitrarily fixed, then it is not possible to obtain exact frequency equations as described above.

For such problems, you can successfully apply an approximate method of obtaining frequency equations based on the decomposition method developed in the works of Professor G.I.Pshenichniy [74] for static problems.

Let us consider a number of problems of oscillation of flat rectangular elements under arbitrary boundary conditions along the edges of an element in order to determine the natural oscillation frequencies by the decomposition method.

We present the formulation of the method in the case of a flat element, when the material of the element is elastic. In the future, the method will be applied to elements of a high elastic material.

In the case of a flat element made of an elastic material, an approximate fourth order transverse-oscillation equation is written as

$$\Delta^2 W - D_0 \frac{\partial^2}{\partial t^2}\Delta W + D_1 \frac{\partial^4 W}{\partial t^4} + D_2 \frac{\partial^2 W}{\partial t^2} = 0, \tag{11}$$

where the coefficients are determined by the geometry and material properties of the flat element.

The solution of equation (11) will be sought in the form

$$W = \exp\left(i\frac{b}{h}\right) W_0(x, y) \tag{12}$$

Substituting (4.6.2) into equations (4.6.1), for W_0 we obtain the equation

$$\Delta^2 W_0 + D_0\left(\frac{b}{h}\right)^2 \xi^2 \Delta W_0 + \xi^2\left(\frac{b}{h}\right)^2\left[D_1\left(\frac{b}{h}\right)^2 \xi^2 - D_2\right] W_0 = 0 \quad (13)$$

To use the decomposition method, it is more convenient to introduce new independent and dependent variables.

$$\alpha = \frac{\pi}{l_1}x; \qquad \beta = \frac{\pi}{l_2}y; \qquad W_0 = \frac{l_1^4}{\pi^4}v;$$
$$\lambda = \frac{l_1}{l_2}; \qquad \lambda_1 = \frac{l_1}{\pi h} \tag{14}$$

In variables (14), equation (13) takes the form

$$\left[\frac{\partial^4 v}{\partial \alpha^4} + 2\lambda^2 \frac{\partial^4 v}{\partial \alpha^2 \partial \beta^2} + \lambda^4 \frac{\partial^4 v}{\partial \beta^4}\right] + \lambda_1^2 D_0 \left(\frac{b}{h}\right)^2 \xi^2 \left[\frac{\partial^2 v}{\partial \alpha^2} + \lambda^2 \frac{\partial^2 v}{\partial \beta^2}\right] + \lambda_1^4 \left(\frac{b}{h}\right)^2 \xi^2 \times$$
$$\times \left[D_1\left(\frac{b}{h}\right)^2 \xi^2 - D_2\right] v = 0 \tag{15}$$

The method of decomposition in the theory of oscillations in general formulation reduces to the following.

The formulation of auxiliary problems is formulated.

1. Find a solution to the equation

$$\frac{\partial^4 v_1}{\partial \alpha^4} = f^{(1)}(\alpha, \beta) \tag{16}$$

under boundary conditions

$$L_1(\alpha, \beta) = 0; \qquad L_2(\alpha, \beta) = 0; \qquad (\alpha = 0; \pi) \tag{17}$$

2. Find a solution to the equation

$$\lambda^4 \frac{\partial^4 v_2}{\partial \beta^4} = f^{(2)}(\alpha, \beta) \tag{18}$$

under boundary conditions

$$L_3(\alpha, \beta) = 0; \qquad L_4(\alpha, \beta) = 0; \qquad (\beta = 0; \pi) \tag{19}$$

The boundary conditions at the edges of the plate depend on the conditions of its fixation or on the free edge from stresses.

Rest of the equation (15)

$$2\lambda \frac{\partial^4 v_3}{\partial \alpha^2 \partial \beta^2} + \lambda D_0 \left(\frac{b}{h}\right)^2 \xi^2 \left(\frac{\partial^2 v_3}{\partial \alpha^2} + \lambda^2 \frac{\partial^2 v_3}{\partial \beta^2}\right) + \lambda_1^4 D_0 \left(\frac{b}{h}\right)^2 \xi^2 \left[D_1 \left(\frac{b}{h}\right)^2 \xi^2 - D_2\right] v_3 + \\ + f^{(1)}(\alpha, \beta) + f^{(2)}(\alpha, \beta) = 0, \tag{20}$$

where $f^{(j)}(\alpha, \beta)$ arbitrary functions the forms of which depend on the boundary-value problems.

Following the decomposition method, we assume that

$$v_3 = \frac{1}{2}[v_1 + v_2] \tag{21}$$

and the condition must be met at given points on the flat element.

The general solutions of the auxiliary problems equations (16) and (18) are

$$\begin{aligned} v_1 &= f_1(\alpha, \beta) + \frac{\alpha^3}{6}\varphi_1(\beta) + \frac{\alpha^2}{2}\varphi_2(\beta) + \alpha\varphi_3(\beta) + \varphi_4(\beta); \\ v_1 &= f_1(\alpha, \beta) + \frac{\beta^3}{6}\psi_1(\alpha) + \frac{\beta^2}{2}\psi_2(\alpha) + \beta\psi_3(\alpha) + \psi_4(\alpha); \end{aligned} \tag{22}$$

where φ_j, ψ_j arbitrary functions of the arguments and are determined from the boundary conditions (17) и (19).

In the following, arbitrary functions in the general form will be represented as

$$f^{(j)}(\alpha, \beta) = \sum_{n=1}^{\infty} \sum_{j=1}^{\infty} a_{n,m}^{(j)} \sin(\alpha n) \sin(\beta m), \tag{23}$$

where $a_{n,m}^{(j)}$ arbitrary constants, and functions $f_j(\alpha, \beta)$ in common solutions (22) are equal

$$f_1(\alpha, \beta) = \sum_{n=1}^{\infty} \sum_{j=1}^{\infty} \frac{a_{n,m}^{(j)}}{n^4} \sin(\alpha n) \sin(\beta m);$$

$$f_2(\alpha,\beta)=\sum_{n=1}^{\infty}\sum_{j=1}^{\infty}\frac{a_{n,m}^{(2)}}{m^4}\sin(\alpha n)\sin(\beta m). \qquad (24)$$

Using private solutions of problems under given boundary conditions and using approximate representations (21), to find the unknowns $a_{n,m}^{(j)}$ we obtain a homogeneous linear system of algebraic equations whose nontrivial solution leads to the frequency equation.

We illustrate the decomposition method on a number of particular boundary-value problems of the oscillation of a flat element.

Problem 1. We consider the simplest problem when all edges are hinged. This problem was solved by the direct method (13) and the frequency equation (14) was obtained, where it is necessary to set the relaxation time to infinity.

Boundary conditions have the form

$$\begin{aligned} v_1 &= \frac{\partial^2 v_1}{\partial\alpha^2}=0 \qquad (\alpha=0;\pi); \\ v_2 &= \frac{\partial^2 v_2}{\partial\beta^2}=0 \qquad (\beta=0;\pi), \end{aligned} \qquad (25)$$

satisfying which general solutions (22), we get

$$v_1=f_1(\alpha,\beta); \qquad \lambda^4 v_2=f_2(\alpha,\beta) \qquad (26)$$

or private solutions are equal

$$\varphi_j(\beta)=\psi_j(\alpha=0) \qquad j=(1,\ldots,4)$$

Satisfying solution (26) to conditions (21) and equation (20), for the frequency ξ we again obtain equation (14).

Thus, an approximate decomposition method gives the same result as the exact direct method. Consequently, the decomposition method can be applied with a sufficient degree of reliability in the solution of other boundary-value problems.

Problem 2. A rigidly fixed plate on the edges. Boundary conditions have the form

$$\begin{aligned} v_1 &= \frac{\partial v_1}{\partial\alpha}=0 \qquad (\alpha=0;\pi); \\ v_1 &= \frac{\partial v_2}{\partial\beta}=0 \qquad (\beta=0;\pi); \end{aligned} \qquad (27)$$

Using general solutions (22) and boundary-value solutions (27), for the unknown quantities v_1, v_2 get expressions

$$v_1=f_1(\alpha,\beta)-\frac{\alpha^3}{\pi^2}\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{a_{n,m}^{(1)}}{n^3}\left[1+(-1)^n\right]\sin(\beta m)+$$

$$+\frac{\alpha^2}{\pi}\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{a_{n,m}^{(1)}}{n^3}\left[2+(-1)^n\right]\sin(\beta,m)-\alpha\frac{\alpha^3}{\pi^2}\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{a_{n,m}^{(1)}}{n^3}\sin(\beta m);$$

$$v_2 = f_2(\alpha,\beta) - \frac{\beta^3}{\pi^2}\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{a_{n,m}^{(1)}}{m^3}\left[1+(-1)^m\right]\sin(\alpha m) +$$
$$+\frac{\beta^2}{\pi}\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{a_{n,m}^{(1)}}{m^3}\left[2+(-1)^m\right]\sin(\alpha m) - \beta\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{a_{n,m}^{(1)}}{m^3}\sin(\alpha n). \quad (28)$$

We confine ourselves to the first coefficients in the series of arbitrary functions (23) and the condition $v_1, = v_2; (\alpha,\beta) = \frac{\pi}{2}$, we get a system of algebraic equations

$$\left[a_{1,1}^{(1)} + \lambda^{-4}a_{1,1}^{(2)}\right]\left\{\lambda^2\left(1-\frac{2}{\pi}\right) + \frac{(2-\gamma)}{2}\lambda_1^2\xi^2\left[\frac{2}{\pi} - 1 + \lambda^2\left(\frac{\pi}{4}-1\right)\right] +\right.$$
$$\left.+\frac{1}{2}\lambda_1^4\xi^2\left[\frac{(7-8v)}{8}\xi^2 - \frac{3(1-v)}{2}\right]\left(1+\frac{\pi}{4}\right) + \frac{1}{2}\right\} = 0;$$
$$a_{1,1}^{(1)} = \lambda^{-4}a_{1,1}^{(2)} \quad (29)$$

Nontrivial solution of system (29) to the frequency equation

$$\lambda_1^4\frac{(7-8v)}{8}\xi^4 - \frac{\lambda_1^2}{2}\left[3-(1-v)\lambda_1^2 + (2-v)\left(2-\frac{1}{\pi}\right)\left(1+\lambda^6\right)\right]\xi^2 +$$
$$+\left[2\lambda^2\left(1-\frac{1}{\pi}\right) + \left(1+\lambda^4\right)\right] = 0 \quad (30)$$

<u>Problem 3.</u> The edges of the plate $\beta = 0; \beta = \pi$ are rigidly fixed and the edges $\alpha = 0; \alpha = \pi$ are free from stresses i.e. we have boundary conditions

$$\frac{\partial^2 v_1}{\partial\alpha^2} + Q_0 v_1 = 0; \frac{\partial^3 v_1}{\partial\alpha^3} = 0, (\alpha = 0; \pi)$$
$$Q_0 = \left(\frac{3-2v}{7-4v}\right)\left[2\lambda^2\frac{\partial^2}{\partial\beta^2} + \lambda_1^2\xi^2\right]; \quad (31)$$
$$v_2 = \frac{\partial v^2}{\partial\beta} = 0 \ (\beta = 0; \pi)$$

The solution of the problem to determine v_2 has the form (2 8).

To find the unknown function v_1 from boundary conditions

$$\frac{\partial^3 v_1}{\partial\alpha^3} = 0 \text{ at } \alpha = 0; \pi$$

we obtain

$$\varphi_1 = -\left.\frac{\partial^3 f_1}{\partial a^3}\right|_{a=0;} \varphi_1 = -\left.\frac{\partial^3 f_1}{\partial a^3}\right|_{a=\pi}; \quad (32)$$

which can be fulfilled at $n = 2q$ that is odd values of unknowns $a_{n,m}^{(1)}$ must be set to zero.

Conditions (31) at $\alpha = 0; \pi$ lead to the system

$$[\pi\varphi_1 + \varphi_2] + \left(\frac{3-2v}{7-4v}\right)\left[\left(\frac{\pi^3}{6}\frac{\partial^2\varphi_1}{\partial\beta^2} + \frac{\pi^2}{2}\frac{\partial^2\varphi_2}{\partial\beta^2} + \pi\frac{\partial^2\varphi_3}{\partial\beta^2} + \frac{\partial^2\varphi_4}{\partial\beta^2}\right) + \right.$$

$$\left. + \lambda_1^2\xi^2\left(\frac{\pi^3}{6}\varphi_1 + \frac{\pi^2}{2}\varphi_2\pi\varphi_3 + \varphi_4\right)\right] = 0$$

$$\varphi_2 = -\left(\frac{3-2v}{7-4v}\right)\left(\frac{\partial^2\varphi_4}{\partial\beta^2} + \lambda_1^2\xi^2\varphi_4\right) \tag{33}$$

Two equations (33) connect three unknown functions. Since we are looking for private solutions of problems without limiting the generality, the unknown function φ_3 can be put equal to $\varphi_3 = 0$.

уравнения (33) связывают три неизвестные функции. Так как ищем частные решения задач, то не ограничивая общности, неизвестную функцию φ_3 можно положить равной $\varphi_3 = 0$.

From the system (33) we get the equation for φ_4 :

$$\frac{\partial^4\varphi_4}{\partial\beta^4} + 2\lambda_1^2\xi^2\frac{\partial^2\varphi_4}{\partial\beta^2} + \lambda_1^4\xi^4\varphi_4 = -\frac{2}{\pi}\left(\frac{7-4v}{3-2v}\right)^2\left\{\pi\left[1 + \left(\frac{3-2v}{7-4v}\right)\frac{\pi^2}{6}\lambda_1^2\xi^2\right]\frac{\partial^3 f_1}{\partial\alpha^2}\bigg|_{\alpha=0} + \right.$$

$$\left. + \frac{\pi^3}{6}\left(\frac{3-2v}{7-4v}\right)\frac{\partial^5 f_1}{\partial\alpha^3\partial\beta^2}\bigg|_{\alpha=\pi}\right\}, \tag{34}$$

whose particular solution is equal to

$$\varphi_4 = \sum_{q=1}^{\infty}\sum_{m=1}^{\infty} a_{2q,m}^{(1)} A_{q,m}^{(1)} \sin(\beta m), \tag{35}$$

где

$$A_{q,m}^{(1)} = \frac{(m^2-1)}{2q}\left(m^4 - 2m^2\lambda_1^2\xi^4\right)^{-1} \tag{36}$$

Restricting to the first components $a_{2,1}^{(1)}; a_{1,1}^{(2)}$, as in the previous problem, we obtain the frequency equation

$$\frac{\pi^2}{192}\lambda_1^4(7-8v)\xi^4 - \left\{\left(\frac{2-v}{2}\right)\lambda_1^2\left[\left(\frac{\pi^2}{24}-1\right) + \frac{\lambda^2\pi^2\left(2-\frac{\pi}{4}-\frac{2}{\pi}\right)}{24\left(1-\frac{\pi}{4}\right)}\right] - \frac{3(1-v)}{48}\lambda_1^4\pi^2\right\}\xi^2 +$$

$$+ \left\{\lambda^2\left[\frac{\pi^2\left(1-\frac{2}{\pi}\right)}{24\left(1-\frac{\pi}{4}\right)} - 1\right] + \left[1 - \lambda^4\frac{\pi^3}{48\left(1-\frac{\pi}{4}\right)}\right]\right\} = 0 \tag{37}$$

Problem 4. The edges of a plate $(\beta = 0;\pi);\ \alpha = 0$ rigidly clamped and the edge $\alpha = \pi$ is free from stress.

In this problem the desired function v_2 is determined in the previous problems and v_1 is equal to

$$v_1 = f_1(\alpha,\beta) + \frac{a^3}{6}\varphi_1(\beta) + \frac{a^2}{2}\varphi_2(\beta) + a\varphi_3\beta;$$
$$\varphi_4 = 0;\ \varphi_1 = -\left.\frac{\partial^3 f_1}{\partial a^3}\right|_{a=\pi};\qquad \varphi_3 = -\left.\frac{\partial f_1}{\partial a}\right|_{a=0}; \tag{38}$$

where

$$\frac{\pi^2}{2}\left(\frac{3-2v}{7-4v}\right)\frac{\partial^2\varphi_2}{\partial\beta^2} + \left[1 + \left(\frac{3-2v}{7-4v}\right)\frac{\pi^2}{2}\lambda_1^2\xi^2\right]\varphi_2 = \left[\pi\left.\frac{\partial^3 f_1}{\partial a^3}\right|_{a=\pi} + \frac{\pi^3}{6}\left(\frac{3-2v}{7-4v}\right)\left.\frac{\partial^5 f_1}{\partial a^3\partial\beta^2}\right|_{a=\pi} + \right.$$
$$\left. + \left(\frac{3-2v}{7-4v}\right)\pi\left.\frac{\partial^3 f_1}{\partial a^3}\right|_{a=\pi} + \left(\frac{3-2v}{7-4v}\right)\pi\lambda_1^2\xi^2\left.\frac{\partial f_1}{\partial\alpha}\right|_{a=0}\right] \tag{39}$$

As in the previous problems, we obtain the frequency equation

$$\lambda^2\left[\left(1 + \frac{\pi}{2} - B_1 + C_1\left(1 - \frac{2}{\pi}\right)\right] + \frac{(2-v)}{2}\lambda_1^2\xi^2\left\{\left[\left(B_1 - \frac{\pi}{2} - 1\right) - \right.\right.$$
$$\left. - C_1\left(1 + \frac{\pi}{2}\right)\right] + \lambda^2\left[-\left(1 - \frac{\pi}{2} - \frac{\pi^3}{48} + \frac{\pi^2}{8}B_1\right) + C_1\left(\frac{2}{\pi} - 1\right)\right]\right\} +$$
$$+ \lambda_1^4\xi^2\left[\left(\frac{7-8v}{8}\right)\xi^2 - \frac{3}{2}(1-v)\right]\left[\left(1 - \frac{\pi^3}{48} - \frac{\pi}{2} + \frac{\pi^2}{48}B_1\right) + \right.$$
$$\left. + C_1\left(1 - \frac{\pi}{4}\right)\right] + \left(1 + C_1\lambda^4\right) = 0; \tag{40}$$

where B_1, C_1 are equal to

$$B_1 = \left[\frac{\pi}{4} - \frac{\pi^3}{6}\left(\frac{3-2v}{7-4v}\right) - \pi\left(\frac{3-2v}{7-4v}\right) + \pi\lambda_1^2\xi^2\left(\frac{3-2v}{7-4v}\right)\right]\times$$
$$\times\left[1 + \frac{\pi^2}{2}\left(\frac{3-2v}{7-4v}\right)\lambda_1^2\xi^2 - \frac{\pi^2}{2}\left(\frac{3-2v}{7-4v}\right)\right]^{-1}; \tag{41}$$
$$C_1 = \left(1 - \frac{\pi}{2} - \frac{\pi^3}{48} + \frac{\pi^2}{8}B_1\right)\left(1 - \frac{\pi}{4}\right)^{-1}$$

Frequency equation (40) defines three frequencies unlike the previous ones which is apparently, connected with the fact that the edge $a = \pi$ is free from stresses and the waves are reflected from the rigidly fixed edge $a = 0$.

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ДЕКОМПОЗИЦИЯ ӘДІСІМЕН ШЫҒАРЫЛҒАН КЕЙБІР ТЕРБЕЛІС ЕСЕБІНІҢ ШЕШІМДЕРІН САНДЫҚ ТАЛДАУ

Аннотация: Тік бұрышты пішіндегі жазық пластинкалар құрылыс конструкцияларының және ғимараттарының негізгі элементтерінің бірі болып табылады. Тік бұрышты жазық элементтердің қолданбалы тербеліс есептерін шешу кезінде шеттік есептер үшін, жазық элементтердің бастапқы шарттары мен шетіндегі шегаралық шарттарына байланысты әр-түрлі жоғары санатты есептер пайда болады. Тербелістер теориясында меншікті тербелістің жиілігін анықтау, жазық элементтердің еріксіз тербеліс есебің шешу және ондағы гармоникалық толқындардың таралуын зерттеу негізгі кезең болып табылады. Бұл жұмыста жазық элементтердің өзіндік және еріксіз тербелісін, материал элементтерінің қатпарлылығын, тұтқыр реологиялық қасиетін, қоршаған ортаның әсерін, негізінің деформацияға ұшырауы, анизотропиясын және тағы басқа да жасалған зеріттеу шешімдері келтіріледі, себебі көрсетілген факторлардың әсері жазық элементтердің өзіндік және еріксіз тербелісі есебіндегі гармоникалық толқындардың таралу процесін зертеуді айтарлықтай қиындатады.

Түйін сөздер: +өзіндік тербеліс, еріксіз тербеліс, жиіліктік теңдеулері, трансцендenttік теңдеулер, декомпозици әдісі, таралу уақыты, кернеу, пластинка

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ЧИСЛЕННЫЙ АНАЛИЗ РЕШЕНИЯ НЕКОТОРЫХ ЗАДАЧ КОЛЕБАНИЯ МЕТОДОМ ДЕКОМПОЗИЦИИ

Аннотация: Плоские пластинки прямоугольной формы являются одними из основных элементов строительных конструкций и сооружений. При решении прикладных задач колебания прямоугольных плоских элементов возникает широкий класс задач колебаний, связанных с различными краевыми задачами: приближёнными уравнениями колебания, различными граничными условиями на краях плоского элемента и начальными условиями. В теории колебания важным моментом является определение частот собственных колебаний, решение задач о вынужденных колебаниях плоского элемента и исследование распространения гармонических волн в них. В данной работе приводятся результаты по исследованию собственных и вынужденных колебаний плоских элементов с учётом слоистости материала элемента, реологических вязких свойств, влияния окружающей среды, деформируемого основания, анизотропии и т.д. Влияние указанных факторов значительно затрудняет исследование задач о собственных и вынужденных колебаниях плоского элемента, о распространении в них гармонических волн.

Ключевые слова: собственная колебания, вынужденная колебания, частотные уравнения, трансцендентные уравнения, метод декомпозиции, время релаксации, напряжения, пластинка

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N E W S
OF THE NATIONAL ACADEMY OF SCIENCES OF THE REPUBLIC OF KAZAKHSTAN
PHYSICO-MATHEMATICAL SERIES
ISSN 1991-346X **https://doi.org/10.32014/2019.2518-1726.5**
Volume 1, Number 323 (2019), 38 – 45

UDC 517.946:517.588
IRSTI 27.29.21; 27.23.25

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THE CONSTRUCTION OF A SOLUTION OF A RELATED SYSTEM OF THE LAGUERRE TYPE

Abstract. The aim of the work is to study the system of Laguerre type obtained from the degenerate of Horn system by direct selection of parameters, as well as using an exponential transformation. Such a system consisting of two partial differential equations of second order is called related to the basic Laguerre system. The difficulties of studying are that if in the ordinary case there is one degenerate of Kummer's equation and only one degenerate hypergeometric function satisfying it, then in the case of two variables there 20 degenerate systems and 20 degenerate hypergeometric functions of two variables satisfying them appear. It is not known how many systems of Laguerre type exist, and with which of the 20 degenerate systems it links to. There is no general method of a research In this work Frobenius-Latysheva's method which is generalized in this case by Zh.N. Tasmambetov is applied to creation of their normal and regular solution depending on Laguerre's polynomial of two variables. The classification of singular curves using rank and antirank is given, as well as basic information about the features of constructing normal-regular solutions of such systems. The main theorem on the existence of four linearly independent partial solutions is proved. Solutions are determined by the degenerate hypergeometric function of M.P. Humbert in the form of normal-regular ranks of two variables depending on Laguerre's polynomials. The conclusions indicate the relationship of such systems with overridden systems and some representations of Laguerre's polynomial of two variables.

Key words: Related, system, Laguerre-type system, Horn system, normal-regular solution, special curves, rank, antirank, overdetermined.

Introduction

The degenerate hypergeometric function is the root of many well-known functions, and through it all orthogonal polynomials of one variable are expressed [1]-[2]. Indeed, if γ is not an integer, is α a negative integer or zero, then the series

$$G(\alpha,\gamma;x)=1+\frac{\alpha}{\gamma}x+\frac{\alpha(\alpha+1)}{2!\gamma(\gamma+1)}x^2+... \qquad (1)$$

representing the degenerate hypergeometric function terminates, and we obtain a polynomial $G(-n,\gamma;x)$ in particular expressing the polynomial of Laguerre. In the theory of orthogonal polynomials, there are several differential equations solutions of which are Laguerre's polynomials and various applications in the problems of mathematical physics, as well as in the theory of the hydrogen atom, etc. [3, c.226]- [4]-[5,115-118]. The generalization of this theory to Laguerre's polynomials of two variables and systems of partial differential equations of the second order, which they satisfy, has not reached this level. The study is complicated by the fact that if in the ordinary case there is only one degenerate hypergeometric equation, then in the case of two variables there are 20 degenerate systems and 20 degenerate hypergeometric functions of two variables satisfying them [6, c.226-230]-[7]. It is not yet known how many systems of the Laguerre type are there and with which of the 20 degenerate systems they are connected. Apparently, this was influenced by the insufficient development of the analytical theory of such systems. Therefore, another direction for studying orthogonal polynomials of two variables as eigenfunctions of linear partial differential operators of the second order was developed [8]-[9]-[10].

In the works of Zh.N.Tasmambetov and A.A.Issenova the system of Horn (Ψ_2) was selected as a binding system and the connection between the degenerate hypergeometric function of Humbert $\Psi_2(\alpha,\gamma,\gamma';x,y)$ and the Laguerre's function of two variables $L_{n,m}^{(\alpha,\beta)}(x,y)$ was studied.

In [11] it was shown that from the system of Horn

$$\left.\begin{aligned} xZ_{xx}+(\gamma-x)Z_x-yZ_y+nZ=0\\ yZ_{yy}+(\gamma-y)Z_y-xZ_x+nZ=0 \end{aligned}\right\} \qquad (2)$$

when $\gamma=\alpha+1,\gamma'=\beta+1,\alpha\neq 0,\beta\neq 0,\lambda=-n$ the basic system of Laguerre is obtained

$$\left.\begin{aligned} xZ_{xx}+(\alpha+1-x)Z_x-yZ_y+nZ=0\\ yZ_{yy}+(\beta+1-y)Z_y-xZ_x+nZ=0 \end{aligned}\right\} \qquad (3)$$

solution of which is a polynomial

$$\Psi_2(-n,\alpha+1,\beta+1;x,y)=\sum_{\mu,\nu=0}^{\infty}\frac{(-n)_{\mu+\nu}}{(\alpha+1)_\mu(\beta+1)_\nu}\cdot\frac{x^\mu}{\mu!}\cdot\frac{y^\nu}{\nu!}. \qquad (4)$$

By analogy, this polynomial is called the generalized Laguerre's polynomials of two variables and is denoted by

$$L_{n,m}^{(\alpha,\beta)}=\Psi_2(-n,\alpha+1,\beta+1;x,y). \qquad (5)$$

Basic information

According to the general theory of systems of the form (2), when the condition of compatibility and integrality is performed [12], it has up to four linearly independent solutions $Z_i\left(i=\overline{1,4}\right)$, and the general solution depends on arbitrary constants and is represented as a sum

$$Z(x,y)=C_1Z_1(x,y)+C_2Z_2(x,y)+C_3Z_3(x,y)+C_4Z_4(x,y) \qquad (6)$$

where $C_i\left(i=\overline{1,4}\right)$ are arbitrary constants, $Z=Z(x,y)$ is a general unknown.

The system has a regular $(0,0)$ singularity and an irregular (∞,∞) singularity. To classify regular and irregular singularities K.Ya. Latysheva used the notion of rank

$$p=1+k,k=\max_{(1\leq s\leq n)}\frac{\beta_s-\beta_0}{s} \qquad (7)$$

introduced by H. Poincare and antirank

$$m=-1-\chi,\chi=\min_{(1\leq s\leq n)}\frac{\pi_s-\pi_0}{s} \qquad (8)$$

introduced by L. Tome.

These concepts were generalized to the case of the studied system of (2) Zh.N.Tasmambetov[13].

If the rank is $p\leq 0$, then the special curve $(x=\infty,y=\infty)$ is regular, when $p>0$ the special curve is irregular. When $m\leq 0$ a special curve $(0,0)$ is regular, and if $m>0$ special is irregular [13].

Definition 1. If the rank $p>0$ and antirank $m\leq 0$, then system (2) has a solution

$$Z(x,y)=\exp Q(x,y)\cdot x^{\rho_i}y^{\sigma_i}\sum_{\mu,\nu=0}^{\infty}A_{\mu,\nu}^{(i)}x^{\mu}y^{\nu},\ A_{0,0}\neq 0, \tag{9}$$

where $\rho_i,\sigma_i\left(i=\overline{1,4}\right), A_{\mu,\nu}^{(i)}(\mu,\nu=0,1,2,...)$ - unknown constants; $Q(x,y)$ - polynomial of two variables

$$Q(x,y)=\frac{\alpha_{p0}}{p}x^p+\frac{\alpha_{0p}}{p}y^p+...+\alpha_{11}xy+\alpha_{10}x+\alpha_{01}y, \tag{10}$$

with unknown coefficients $\alpha_{p0},\alpha_{0p},...,\alpha_{11},\alpha_{10},\alpha_{01}$. The solution of the form (9) is called normal-regular.

If the special curve $(0,0)$ is regular, then the polynomial $Q(x,y)\equiv 0$ and the solution of the system exists in the form of a generalized power series of two variables

$$Z(x,y)=x^{\rho_i}y^{\sigma_i}\sum_{\mu,\nu=0}^{\infty}A_{\mu,\nu}^{(i)}x^{\mu}y^{\nu},\ A_{0,0}\neq 0, \tag{11}$$

where $\rho_i,\sigma_i\left(i=\overline{1,4}\right), A_{\mu,\nu}^{(i)}(\mu,\nu=0,1,2,...)$ - the unknown constants.

The highest degree of the polynomial $Q(x,y)$ is determined by the rank p.

Definition 2. The values of number p determined by the equality (7) are called series order (9) and can be an integer or a fractional number (positive or negative).

CONCLUSION OF THE RELATED SYSTEM OF THE LAGUERRE TYPE AND THE CONSTRUCTION OF ITS SOLUTION

Formulation of the problem

From the system of Horn (Ψ_2) by means of converting

$$Z=\exp\left(\frac{x}{2}+\frac{y}{2}\right)\cdot x^{-\frac{\alpha+1}{2}}y^{-\frac{\beta+1}{2}}\cdot U(x,y) \tag{12}$$

a system of Laguerre type is installed

$$\left.\begin{aligned}&x^2U_{xx}-xyU_y+\left(-\frac{x^2}{4}-\frac{xy}{2}+kx+\frac{1}{4}-\alpha^2\right)\cdot U=0\\&y^2U_{yy}-xyU_x+\left(-\frac{y^2}{4}-\frac{xy}{2}+ky+\frac{1}{4}-\beta^2\right)\cdot U=0\end{aligned}\right\} \tag{13}$$

where $k=(\alpha+\beta+2-2\lambda)/2$ is related with the basic Laguerre system (3).

Such systems belong to the Whitaker-type system [7]. By applying Frobenius-Latysheva method [13] we want to establish distinctive features of the system (12) and construct its normal-regular solution dependent on Laguerre's polynomials of two variables.

MAIN RESULTS

Theorem 1. The system of second order partial differential equations (13) has four linearly independent partial solutions, which are expressed through the degenerate hypergeometric function of M.R. Humbert $\Psi_2(\alpha,\gamma,\gamma';x,y)$ in the form of normal-regular series

$$\begin{aligned}U(x,y)&=\exp\left(-\frac{x}{2}-\frac{y}{2}\right)\cdot x^{\frac{\alpha+1}{2}}y^{\frac{\beta+1}{2}}\cdot\Psi_2(-n,\alpha+1,\beta+1;x,y)=\\&=\exp\left(-\frac{x}{2}-\frac{y}{2}\right)\cdot x^{\frac{\alpha+1}{2}}y^{\frac{\beta+1}{2}}\cdot L_{n,n}^{(\alpha,\beta)}(x,y)\end{aligned} \tag{14}$$

dependent on the Laguerre's polynomial of two variables

$$L_{n,n}^{(\alpha,\beta)}(x,y)=1-\frac{n}{1!(\alpha+1)}x-\frac{n}{1!(\beta+1)}y+\frac{n(n-1)}{1!(\alpha+1)(\beta+1)}xy+\frac{n(n-1)}{1!(\alpha+1)(\alpha+2)}x^2+$$
$$+\frac{n(n-1)}{1!(\beta+1)(\beta+2)}y^2+\ldots+(-1)^n\frac{n(n-1)\ldots1}{n!(\alpha+1)\ldots(\alpha+n)}x^n+ \tag{15}$$
$$+(-1)^n\frac{n(n-1)\ldots1}{n!(\alpha+1)\ldots(\alpha+n-1)(\beta+1)}x^{n-1}y+\ldots+(-1)^n\frac{n(n-1)\ldots1}{n!(\beta+1)\ldots(\beta+n)}y^n$$

Evidence. For the proof Frobenius-Latysheva method is used. Like the degenerate system (2) the system (12) has a regular $(0,0)$ singularity and an irregular (∞,∞) singularity. By highest degrees of independent variables x and y certain subranks: $k_1'=0, k_1''=0$ and rank $p=1+k=1$. Then according to the method of Frobenius-Latysheva for the construction of normal-regular solution of (9), in the system (13) the transformation is correct:

$$U=\exp(\alpha_{10}x+\alpha_{01}y)\Phi(x,y) \tag{16}$$

where α_{10} and α_{01} are uncertain coefficients, which need to be determined from the newly obtained support system.

$$\left.\begin{aligned}&x^2\Phi_{xx}+2\alpha_{10}^2x^2\Phi_x-xy\Phi_y+\left(\left(\alpha_{10}^2-\frac{1}{4}\right)x^2-\left(\alpha_{01}^2-\frac{1}{2}\right)xy+kx+\frac{1}{4}-\frac{\alpha^2}{4}\right)\cdot\Phi=0\\&y^2\Phi_{yy}+2\alpha_{01}^2y^2\Phi_y-xy\Phi_x+\left(\left(\alpha_{01}^2-\frac{1}{4}\right)y^2-\left(\alpha_{10}^2-\frac{1}{2}\right)xy+ky+\frac{1}{4}-\frac{\beta^2}{4}\right)\cdot\Phi=0\end{aligned}\right\} \tag{17}$$

By equating coefficients to zero at the highest degrees of independent variables x^2 and y^2 at unknown $\Phi(x,y)$, we define a system of characteristic equations

$$\begin{aligned}&b_{10}^{(1)}(\alpha_{10},\alpha_{01})=\alpha_{10}^2-\frac{1}{4}=0,\\&b_{01}^{(2)}(\alpha_{10},\alpha_{01})=\alpha_{01}^2-\frac{1}{4}=0.\end{aligned} \tag{18}$$

This shows the fulfillment of the first necessary condition for the existence of a normal-regular solution (9) connected with the definition of the unknown coefficients of the $Q(x,y)$ polynomial [13].

Theorem 2. Equality (18) is required for a supporting system to have at least one solution of the form (9).

The system (17) has four root pairs:

$$\begin{aligned}&\left(\alpha_{10}^{(1)},\alpha_{01}^{(1)}\right)=\left(\frac{1}{2},\frac{1}{2}\right),\left(\alpha_{10}^{(1)},\alpha_{01}^{(2)}\right)=\left(\frac{1}{2},-\frac{1}{2}\right),\\&\left(\alpha_{10}^{(2)},\alpha_{01}^{(1)}\right)=\left(-\frac{1}{2},\frac{1}{2}\right),\left(\alpha_{10}^{(2)},\alpha_{01}^{(2)}\right)=\left(-\frac{1}{2},-\frac{1}{2}\right),\end{aligned} \tag{19}$$

defining four polynomials of the first degree of the form (10), since the rank of the system is equal to one:

$$Q_i(x,y)=\alpha_{10}^{(i)}x+\alpha_{01}^{(i)}y,\ i=\overline{1,4}.$$

Four $\left(\alpha_{10}^{(l)},\alpha_{01}^{(l)}\right), l=1,2$ pairs in (18) define four systems from the auxiliary system (18). Each of them can have up to four linearly independent particular solutions. Thus, the initial system should have up to 16 private solutions. However, a detailed study shows that out of the four systems, only the system

$$\left.\begin{aligned} x^2\Phi_{xx}+x^2\Phi_x-xy\Phi_y+\left(kx+\frac{1}{4}-\frac{\alpha^2}{4}\right)\cdot\Phi=0\\ y^2\Phi_{yy}+y^2\Phi_y-xy\Phi_x+\left(ky+\frac{1}{4}-\frac{\beta^2}{4}\right)\cdot\Phi=0\end{aligned}\right\}\qquad(20)$$

has four linearly independent particular solutions. All of them are expressed through degenerate hypergeometric function of Humbert $\Psi_2(\alpha,\gamma,\gamma';x,y)$.

Indeed, by making up the system of characteristic functions of the system (20) we make sure that the system of defining equations with respect to the singularity $(0,0)$

$$\left.\begin{aligned} f_{00}^{(1)}(\rho,\sigma)=\rho(\rho-1)+\frac{1}{4}-\frac{\alpha^2}{4}=0,\\ f_{00}^{(2)}(\rho,\sigma)=\sigma(\sigma-1)+\frac{1}{4}-\frac{\beta^2}{4}=0,\end{aligned}\right\}\qquad(21)$$

has four pairs of roots:

$$\begin{aligned}(\rho_1,\sigma_1)=\left(\frac{1}{2}+\frac{\alpha}{2},\frac{1}{2}+\frac{\beta}{2}\right),(\rho_1,\sigma_2)=\left(\frac{1}{2}+\frac{\alpha}{2},\frac{1}{2}-\frac{\beta}{2}\right),\\ (\rho_2,\sigma_1)=\left(\frac{1}{2}-\frac{\alpha}{2},\frac{1}{2}+\frac{\beta}{2}\right),(\rho_2,\sigma_2)=\left(\frac{1}{2}-\frac{\alpha}{2},\frac{1}{2}-\frac{\beta}{2}\right).\end{aligned}\qquad(22)$$

This shows the fulfillment of the second necessary condition.

Theorem 3. In order for the system (13) to have a normal-regular solution of the form (9), it is necessary for the pair (ρ,σ) to be the root of the defining equations with respect to the $(0,0)$singularity of the form (20) obtained from the auxiliary system (17) by substituting instead of the unknown $Z(x,y)=x^{\rho}\cdot y^{\sigma}$.

The existence of four pairs of roots (22) ensures the existence of four linearly independent particular solutions of the system (20) in the form of generalized power series (12). Since, the values of ρ and σ are defined, it remains to find the unknown constants $A_{\mu,\nu}(\mu,\nu=0,1,2,...)$ with the help of a system of recurrent sequences

$$\sum A_{m-\mu,n-\nu}^{(i)}\cdot f_{\mu,\nu}^{(i)}(\rho+m-\mu,\sigma+n-\nu),(m,n=0,1,2,...;i=1,2;\mu,\nu=0,1,2,...).$$

Thus, the constructed particular solutions of (19) have the form of the

$$\Phi_1(x,y)=x^{\frac{\alpha+1}{2}}y^{\frac{\beta+1}{2}}\cdot\Psi_2\left(\frac{\alpha+\beta}{2}+1-k,\alpha+1,\beta+1;x,y\right)$$

$$\Phi_2(x,y)=x^{\frac{\alpha+1}{2}}y^{\frac{1-\beta}{2}}\cdot\Psi_2\left(\frac{\alpha-\beta}{2}+1-k,\alpha+1,1-\beta;x,y\right)$$

$$\Phi_3(x,y)=x^{\frac{1-\alpha}{2}}y^{\frac{\beta+1}{2}}\cdot\Psi_2\left(\frac{\beta-\alpha}{2}+1-k,1-\alpha,\beta+1;x,y\right)$$

$$\Phi_4(x,y)=x^{\frac{1-\alpha}{2}}y^{\frac{1-\beta}{2}}\cdot\Psi_2\left(\frac{-\alpha-\beta}{2}+1-k,1-\alpha,1-\beta;x,y\right)$$

Considering $k=(\alpha+\beta+2-2\lambda)/2$, these solutions are represented in the form

$$\Phi_1(x,y)=x^{\frac{\alpha+1}{2}}y^{\frac{\beta+1}{2}}\cdot\Psi_2(-n,\alpha+1,\beta+1;x,y)$$

$$\Phi_2(x,y)=x^{\frac{\alpha+1}{2}}y^{\frac{1-\beta}{2}}\cdot\Psi_2(-n-\beta,\alpha+1,1-\beta;x,y)$$
$$\Phi_3(x,y)=x^{\frac{1-\alpha}{2}}y^{\frac{\beta+1}{2}}\cdot\Psi_2(-n-\alpha,1-\alpha,\beta+1;x,y) \quad (23)$$
$$\Phi_4(x,y)=x^{\frac{1-\alpha}{2}}y^{\frac{1-\beta}{2}}\cdot\Psi_2(-n-\alpha-\beta,1-\alpha,1-\beta;x,y)$$

Hence, it is not difficult to notice that the system solution we are interested in (20):

$$\Phi_1(x,y)=x^{\frac{\alpha+1}{2}}y^{\frac{\beta+1}{2}}\cdot\Psi_2(-n,\alpha+1,\beta+1;x,y)=x^{\frac{\alpha+1}{2}}y^{\frac{\beta+1}{2}}\cdot L_{n,n}^{(\alpha,\beta)}(x,y) \quad (24)$$

and the remaining solutions will not be considered in the future.

The fulfillment of two necessary conditions ensures the existence of a normal-regular solution (14), dependent on the Laguerre's polynomial of two variables (15). The theorem is proved.

The General solution of the system (12) on the basis of (6), taking into account formulas (23), is presented as

$$U(x,y)=C_1U_1(x,y)+C_2U_2(x,y)+U_3Z_3(x,y)+C_4U_4(x,y)=$$
$$=C_1\exp\left(-\frac{x}{2}-\frac{y}{2}\right)\Phi_1(x,y)+C_2\exp\left(-\frac{x}{2}-\frac{y}{2}\right)\Phi_2(x,y)+$$
$$+C_3\exp\left(-\frac{x}{2}-\frac{y}{2}\right)\Phi_3(x,y)+C_4\exp\left(-\frac{x}{2}-\frac{y}{2}\right)\Phi_4(x,y),$$

where $C_i\left(i=\overline{1,4}\right)$ - are arbitrary constants.

On the basis of the above reasoning, some statements can be made.

Theorem 4. The system (16) with respect to $\Phi(x,y)$, obtained from (13) by conversion

$$U(x,y)=\exp Q(x,y)\cdot\Phi(x,y) \quad (25)$$

has the same rank as the system (13).

Indeed, since the rank of the system (12) is equal to one, we present transformations (25) in the form of (15) and obtain a system with respect to $\Phi(x,y)$, where the rank is $p=1$. The proof of theorem for the General case is given in the monograph [12].

Theorem 5. The system (13) for which $p>0$, $m\le 0$ has normally regular solution (14), which is expressed through the generalized Laguerre's polynomial of two variables and the right-hand side (14) converges near the singularity $(x=0,y=0)$.

The system (13) is said to be related the system of the Laguerre type. As we have seen, its solutions are also expressed through the degenerate hypergeometric function of M.R. Humbert $\Psi_2(\alpha,\gamma,\gamma';x,y)$ in the form of normal-regular series (14) dependent on the Laguerre's polynomial of two variables (15).

Conclusion: Thus, using the transformation (12), we have established the form of a system of Laguerre's type (3) related to the main system. The application of the Frobenius-Latysheva's method allowed us to construct normal-regular solutions of the derived related system (13) near the singularity $(0,0)$. Generalized Laguerre polynomials also have representations through other hypergeometric functions of two variables [14, p.358].

The limit transition formula is fair [15]

$$\lim_{\alpha\to\infty}L_{m,n}^{(\alpha,\beta,\gamma)}\left(\frac{x}{\alpha},\frac{y}{\alpha}\right)=L_m^{(\beta-1)}(x)\cdot L_n^{(\gamma-1)}(y).$$

Formula (24) can be similarly represented using the same limit transition as a product of Laguerre polynomials in variables x and y.

In the work [16, p. 6-17] the connection of considered systems with the overdetermined systems, studied in the works of Tajik Mathematical School [17] - [18] - [19] was indicated.

The research in this work can be extended to the case of three variables. The connection of the generalized Laguerre's polynomials of one and two variables with generalized hypergeometric functions [20], [21] of many variables was considered in [14] - [15], [22]. However, for this case the main theorem 1 and theorems 2–5 presented here haven't been proved yet. Also, the question of the computational application of special functions of several variables, as in the monograph [23] hasn't been touched upon. Following [24], it is necessary to develop a numerical method for calculating the values of the degenerate hypergeometric Humbert $\Psi_2(\alpha,\gamma,\gamma';x,y)$ functions through the products of Laguerre polynomials in variables and using our formula (24). The problem of the asymptotic expansion, given in [25], is also important when studying the properties of special functions of several variables. We have obtained an asymptotic expansion near the origin $(0,0)$.

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ЛАГЕРРА ТЕКТЕС ТУЫСТАС ЖҮЙЕНІҢ ШЕШІМДЕРІН ТҰРҒЫЗУ

Аннотация. Жұмыстың мақсаты – Горнның туындалған жүйесінен параметрлерді тікелей таңдау және экспоненциал түрлендіру көмегімен алынған Лагерра текті жүйені зерттеу. Мұндай екінші ретті дербес туындылы екі теңдеулерден тұратын дифференциалдық теңдьеулер жүйесін біз, Лагерра текті негізгі жүйемен туыстас деп атадық. Аталған жүйелерді зерттеудің қиындығы мынада: егер жай дифференциалдық теңдеулер жағдайында Куммердің бір туындалған теңдеуі бар және оны қанағаттандыратын бір ғана гипергеометриялық туындалған функциясы бар болса, онда екі айнымалы жағдайында 20 туындалған жүйелер пайда болады және оларды қанағаттандыратын 20 туындалған гипергеометриялық функциялар белгілі. Әзірге, Лагерра тектес қанша жүйелер бар екендігі және олардың туындалған жүйелердің қайсысымен байланыста екендігі белгісіз. Жалпыға ортақ зерттеу әдісі жоқ. Ұсынылған жұмыста екі айнымалының Лагерра көпмүшелігіне тәуелді қалыпты-регуляр шешімдер тұрғызу үшін, екі айнымалы жағдайына Ж.Н.Тасмамбетов жалпылаған Фробениус-Латышева әдісі пайдаланылады. Ранг және антиранг түсініктерін пайдаланып, ерекше қисықтардың классификациясы жасалған және мұндай жүйелердің қалыпты-регуляр шешімдерін тұрғызуға қатысты негізгі түсініктер келтірілген. Төрт сызықты-тәуелсіз дербес шешімдердің бар болатындығы туралы негізгі теорема дәлелденген. Ол дербес шешімдер Лагерраның екі айнымалының көпмүшелігіне тәуелді М.П.Гумберттің туындалған гипергеометриялық функциясы арқылы өрнектелген қалыпты-регуляр қатар арқылы анықталады. Қорытындысында, зерттелген жүйенің артығымен анықталған жүйелермен байланысы және екі айнымалының Лагерра көпмүшелігінің кейбір басқаша өрнектелуі келтірілген.

Түйін сөздер: туыстас, жүйе, Лагерра текті жүйе, Горн жүйесі, қалыпты-регуляр шешім, ерекше қисықтар, ранг, антиранг, артығымен анықталған.

УДК 517.946:517.588

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ПОСТРОЕНИЯ РЕШЕНИЯ РОДСТВЕННОЙ СИСТЕМЫ ТИПА ЛАГЕРРА

Аннотация. Целью работы является изучение системы типа Лагерра, полученной из вырожденной системы Горна непосредственным подбором параметров, а также с помощью экспоненциального преобразования. Такая система, состоящая из двух дифференциальных уравнений в частных производных второго порядка, нами названа родственной с основной системой типа Лагерра. Трудности изучения состоят в том, что если в обыкновенном случае имеет место одно вырожденное уравнение Куммера и только одна вырожденная гипергеометрическая функция, удовлетворяющая ему, то в случае двух переменных появляются 20 вырожденных систем и 20 вырожденных гипергеометрических функций двух переменных удовлетворяющих им. Пока не известно, сколько существуют систем типа Лагерра, и с какими из 20-ти вырожденных систем они связаны. Отсутствует общий метод исследования. В данной работе для построения их нормально-регулярного решения, зависящего от многочлена Лагерра двух переменных, применен обобщенный на этот случай Ж.Н.Тасмамбетовым метод Фробениуса-Латышевой. Приведена классификация особых кривых с помощью ранга и антиранга, а также основные сведения об особенностях построения нормально-регулярных решений таких систем. Доказана основная теорема о существовании четырех линейно-независимых частных решений, которые определяются через вырожденную гипергеометрическую функцию М.П.Гумберта в виде нормально-регулярных рядов зависящих от многочленов Лагерра двух переменных. В выводах указана связь таких систем с переопределенными системами и некоторыми представлениями многочлена Лагерра двух переменных.

Ключевые слова: Родственная, система, система типа Лагерра, система Горна, нормально-регулярное решение, особые кривые, ранг, антиранг, переопределенный

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ISSN 2518-1726 (Online), ISSN 1991-346X (Print)

Редакторы *М. С. Ахметова, Т.А. Апендиев, Д.С. Аленов*
Верстка на компьютере *А.М. Кульгинбаевой*

Подписано в печать 05.02.2019.
Формат 60x881/8. Бумага офсетная. Печать – ризограф.
4,75 п.л. Тираж 300. Заказ 1.

Национальная академия наук РК
050010, Алматы, ул. Шевченко, 28, т. 272-13-18, 272-13-19