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**TELEPARALLEL DARK ENERGY MODEL WITH FERMIONIC
FIELD FOR BIANCHI TYPE I SPACETIME**

Abstract. The search for constituents that can explain the periods of accelerating expansion of the Universe is a fundamental topic in cosmology. In the present work, we consider a model with a fermionic field that is non-minimally coupled to gravity in the framework of teleparallel gravity for Bianchi type I spacetime. Here we determined form the point-like Lagrangian and obtained the corresponding field equations. In order to determine forms of the coupling and potential function of fermionic field for the considered model, we use Noether symmetry approach. In modern cosmology, we often used this approach to determine the unknown function and obtain exact cosmological solutions for considered model. For our model, we obtained of the coupling and fermionic field functions as $F = F_0 \Psi$ and $V = V_0 \psi$. Then, we put these solutions to the field equations, we got the equation depend only on the functions A, B and C. After some mathematical calculations, we found exact solutions for these functions as de Sitter solutions. Finally, we determined the equation of state parameter is equal to -1 . In cosmology this solution gives us dark energy model and which can describes the late-time epoch of the evolution our universe. Thus, here the fermion fields play the role of dark energy.

Key words. Teleparallel dark energy, Bianchi Type I Model, fermionic field, Noethersymmetry approach.

Introduction

In modern cosmology, teleparallel gravity is used to describe the evolution of the universe. For the first time teleparallel theory of gravity (theory of gravity with teleparallelism) was proposed by Einstein. Due to the specific nature of parallel transfers, any calibration theory, including these transformations, will differ in many respects from conventional internal calibration models, and the most significant is the presence of a field tetrad.

On the other hand, field tetrads can be used to determine the linear connectivity of Weizenbock, which is a connectivity defined by torsion but not by the curvature of space. The tetrad field can also be naturally used to introduce a Riemann metric in terms of which the Levi-Civita connectivity can be constructed.

It is important to note that torsion and curvature are connectivity properties and different connectedness can be defined on the same space. Thus, the presence of a nontrivial tetrad field in gauge theory induces both a tele-parallel and a Riemannian structure in space-time. The first obligation of Weizenbock connectivity, the second connectivity Levi-Civita. Due to the universality of the gravitational interaction, it is possible to link these geometric structures in the theory of gravity. Previously, it was considered in [1,2]. Cosmological solutions were obtained within the framework of the theory of the teleparallel of gravity and the Friedman-Robertson-Walker (FRW) metric. The anisotropic model of the Universe for the Bianchi I metric in the framework of the theory of the teleparallel of gravity was considered in the works [3,4].

In modern cosmology, scalar and vector fields, as well as their modifications, such as k-essence, f-essence, g-essence, are used as matter fields. Here we consider fermion fields in the framework of the tele-parallel gravity theory for an anisotropic Bianchi type I. universe. Earlier, cosmological models with fermion fields in the framework of GR for the FRW metric were considered in [5-8]. Also, within the framework of the teleparallel of gravity, models for the FRW metric were considered in [9].

The paper is organized as follows. In Section 2, we present action and equation of motion for this model. The geometrical Noether point symmetries and their connections to the $F(\Psi)$ model is discussed in section 3. Section 4 is analytical solutions for those $F(\Psi)$ models which admit Noether point symmetries. Finally, we draw our main conclusions in section 5.

Action and equation of motion

In this theory, the action is given in the following form

$$S = \int d^4x e \left\{ F(\Psi)T + \frac{i}{2} \left[\bar{\psi} \Gamma^\mu (\bar{\partial}_\mu - \Omega_\mu) \psi - \bar{\psi} (\bar{\partial}_\mu + \Omega_\mu) \Gamma^\mu \psi \right] - V(\Psi) \right\}, \quad (1)$$

where $e = \det(e_\mu^a)$, e_μ^a is a tetrad (vierbein) basis, T is a torsion scalar, and ψ and $\bar{\psi} = \psi^\dagger \gamma^0$ denote the spinor field and its adjoint, with the dagger representing complex conjugation. $F(\Psi)$ and $V(\Psi)$ are generic functions, representing the coupling with gravity and the self-interaction potential of the fermionic field, respectively. In our study, for simplicity, we assume that $F(\Psi)$ and $V(\Psi)$ depend only on functions of the bilinear $\Psi = \bar{\psi} \psi$. In the above action, furthermore, Ω_μ is the spin connection

$\Omega_\mu = -\frac{1}{4} g_{\sigma\nu} \left[\Gamma_{\mu\lambda}^\nu - e_b^\nu \partial_\mu e_\lambda^b \right] \Gamma^\sigma \Gamma^\lambda$ with $\Gamma_{\mu\lambda}^\nu$ denoting the standard Levi-Civita connection and $\Gamma^\mu = e_a^\mu \gamma^a$. The Γ^μ are Dirac matrices.

Together with the action (1), the FRW metric is considered

$$ds^2 = dt^2 - a^2(t)(dx^2 + dy^2 + dz^2),$$

where $a(t)$ is the scale factor of the Universe. Here and then the dot above the letter denotes the derivative in time. This metric describes four-dimensional planar, homogeneous, and isotropic space-time.

In our model we will define field equations for action (1) and Bianchi metrics of type I

$$ds^2 = dt^2 - A^2(t)dx^2 - B^2(t)dy^2 - C^2(t)dz^2, \quad (2)$$

where

$$e = \sqrt{-g} = ABC, T = -2 \left(\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{A}\dot{C}}{AC} + \frac{\dot{C}\dot{B}}{CB} \right), Y = \frac{i}{2} (\bar{\psi} \gamma^0 \dot{\psi} - \dot{\bar{\psi}} \gamma^0 \psi).$$

The Lagrange function for metric (3) can be written as

$$L = 2FC\dot{A}\dot{B} + 2FB\dot{A}\dot{C} + 2FA\dot{C}\dot{B} - \frac{i}{2} ABC (\bar{\psi} \gamma^0 \dot{\psi} - \dot{\bar{\psi}} \gamma^0 \psi) + ABCV, \quad (3)$$

Next, we will use the Euler-Lagrange equations and the zero energy condition to determine the field equations:

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{A}\dot{C}}{AC} + \frac{\dot{C}\dot{B}}{CB} = \frac{1}{2F}\rho_f, \quad (4)$$

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{F}}{F}\left(\frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) = -\frac{1}{2F}p_f, \quad (5)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} + \frac{\dot{F}}{F}\left(\frac{\dot{A}}{A} + \frac{\dot{C}}{C}\right) = -\frac{1}{2F}p_f, \quad (6)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{F}}{F}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right) = -\frac{1}{2F}p_f, \quad (7)$$

$$\dot{\psi} + \frac{1}{2}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right)\psi + iV'\psi\gamma^0 + 2i\left(\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{A}\dot{C}}{AC} + \frac{\dot{C}\dot{B}}{CB}\right)F'\psi\gamma^0 = 0, \quad (8)$$

$$\dot{\bar{\psi}} + \frac{1}{2}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right)\bar{\psi} - iV'\bar{\psi}\gamma^0 - 2i\left(\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{A}\dot{C}}{AC} + \frac{\dot{C}\dot{B}}{CB}\right)F'\bar{\psi}\gamma^0 = 0, \quad (9)$$

where $\rho_f = V$ is the energy density and $p_f = \frac{i}{2}(\bar{\psi}\gamma^0\dot{\psi} - \dot{\bar{\psi}}\gamma^0\psi) - V$ is the pressure of the fluid.

Thus, we obtained the field equations for the anisotropic Universe model in the framework of the theory of body parallel to gravity, where fermionic fields were considered as matter fields. In the following we will investigate various cosmological aspects of these equations.

The Noether symmetry approach

To solve the systems of field equations (4)-(9) we use the Noether symmetry. This approach means that the lie derivative of the point Lagrangian from the vector field X is zero

$$XL = 0. \quad (10)$$

The field vector has the form

$$\begin{aligned} X = & \alpha \frac{\partial}{\partial A} + \beta \frac{\partial}{\partial B} + \gamma \frac{\partial}{\partial C} + \eta_j \frac{\partial}{\partial \psi_j} + \chi_j \frac{\partial}{\partial \psi_j^\dagger} + \dot{\alpha} \frac{\partial}{\partial \dot{A}} + \dot{\beta} \frac{\partial}{\partial \dot{B}} + \\ & + \dot{\gamma} \frac{\partial}{\partial \dot{C}} + \dot{\eta}_j \frac{\partial}{\partial \dot{\psi}_j} + \dot{\chi}_j \frac{\partial}{\partial \dot{\psi}_j^\dagger}, \end{aligned} \quad (11)$$

where

$$\begin{aligned}
 \dot{\alpha} &= \frac{\partial \alpha}{\partial A} \dot{A} + \frac{\partial \alpha}{\partial B} \dot{B} + \frac{\partial \alpha}{\partial C} \dot{C} + \sum_{j=0}^3 \left(\frac{\partial \alpha}{\partial \psi_j} \dot{\psi}_j + \frac{\partial \alpha}{\partial \psi_j^\dagger} \dot{\psi}_j^\dagger \right), \\
 \dot{\beta} &= \frac{\partial \beta}{\partial A} \dot{A} + \frac{\partial \beta}{\partial B} \dot{B} + \frac{\partial \beta}{\partial C} \dot{C} + \sum_{j=0}^3 \left(\frac{\partial \beta}{\partial \psi_j} \dot{\psi}_j + \frac{\partial \beta}{\partial \psi_j^\dagger} \dot{\psi}_j^\dagger \right), \\
 \dot{\gamma} &= \frac{\partial \gamma}{\partial A} \dot{A} + \frac{\partial \gamma}{\partial B} \dot{B} + \frac{\partial \gamma}{\partial C} \dot{C} + \sum_{j=0}^3 \left(\frac{\partial \gamma}{\partial \psi_j} \dot{\psi}_j + \frac{\partial \gamma}{\partial \psi_j^\dagger} \dot{\psi}_j^\dagger \right), \\
 \dot{\eta}_i &= \frac{\partial \eta_i}{\partial A} \dot{A} + \frac{\partial \eta_i}{\partial B} \dot{B} + \frac{\partial \eta_i}{\partial C} \dot{C} + \sum_{j=0}^3 \left(\frac{\partial \eta_i}{\partial \psi_j} \dot{\psi}_j + \frac{\partial \eta_i}{\partial \psi_j^\dagger} \dot{\psi}_j^\dagger \right), \\
 \dot{\chi}_i &= \frac{\partial \chi_i}{\partial A} \dot{A} + \frac{\partial \chi_i}{\partial B} \dot{B} + \frac{\partial \chi_i}{\partial C} \dot{C} + \sum_{j=0}^3 \left(\frac{\partial \chi_i}{\partial \psi_j} \dot{\psi}_j + \frac{\partial \chi_i}{\partial \psi_j^\dagger} \dot{\psi}_j^\dagger \right).
 \end{aligned} \tag{12}$$

By collecting all the terms of equation (12) with coefficients $\dot{A}^2, \dot{B}^2, \dot{C}^2, \dot{A}\dot{B}, \dot{A}\dot{C}, \dot{B}\dot{C}, \dot{A}\dot{\psi}_j, \dot{B}\dot{\psi}_j, \dot{C}\dot{\psi}_j, \dot{A}\dot{\psi}_j^\dagger, \dot{B}\dot{\psi}_j^\dagger, \dot{C}\dot{\psi}_j^\dagger, \dot{A}, \dot{B}, \dot{C}$ and equating them to zero, we obtain the following system of differential equations:

$$C \frac{\partial \beta}{\partial A} + B \frac{\partial \gamma}{\partial A} = 0, \tag{13}$$

$$C \frac{\partial \alpha}{\partial B} + A \frac{\partial \gamma}{\partial B} = 0, \tag{14}$$

$$B \frac{\partial \alpha}{\partial C} + A \frac{\partial \beta}{\partial C} = 0, \tag{15}$$

$$\gamma + C \frac{\partial \alpha}{\partial A} + C \frac{\partial \beta}{\partial B} + B \frac{\partial \gamma}{\partial B} + A \frac{\partial \gamma}{\partial A} + C \frac{F'}{F} \sum_{i=0}^3 (\varepsilon_i \eta_i \psi_i^\dagger + \varepsilon_i \chi_i \psi_i) = 0, \tag{16}$$

$$\beta + B \frac{\partial \alpha}{\partial A} + C \frac{\partial \beta}{\partial C} + A \frac{\partial \beta}{\partial A} + B \frac{\partial \gamma}{\partial C} + B \frac{F'}{F} \sum_{i=0}^3 (\varepsilon_i \eta_i \psi_i^\dagger + \varepsilon_i \chi_i \psi_i) = 0, \tag{17}$$

$$\alpha + C \frac{\partial \alpha}{\partial C} + B \frac{\partial \alpha}{\partial B} + A \frac{\partial \beta}{\partial B} + A \frac{\partial \gamma}{\partial C} + A \frac{F'}{F} \sum_{i=0}^3 (\varepsilon_i \eta_i \psi_i^\dagger + \varepsilon_i \chi_i \psi_i) = 0, \tag{18}$$

$$C \frac{\partial \beta}{\partial \psi_j} + B \frac{\partial \gamma}{\partial \psi_j} = 0, \quad C \frac{\partial \alpha}{\partial \psi_j} + A \frac{\partial \gamma}{\partial \psi_j} = 0, \quad B \frac{\partial \alpha}{\partial \psi_j} + A \frac{\partial \beta}{\partial \psi_j} = 0, \tag{19}$$

$$C \frac{\partial \beta}{\partial \psi_j^\dagger} + B \frac{\partial \gamma}{\partial \psi_j^\dagger} = 0, \quad C \frac{\partial \alpha}{\partial \psi_j^\dagger} + A \frac{\partial \gamma}{\partial \psi_j^\dagger} = 0, \quad B \frac{\partial \alpha}{\partial \psi_j^\dagger} + A \frac{\partial \beta}{\partial \psi_j^\dagger} = 0, \quad (20)$$

$$\sum_{i=0}^3 \left(\frac{\partial \eta_j}{\partial A} \psi_i^\dagger - \frac{\partial \chi_j}{\partial A} \psi_i \right) = 0, \quad \sum_{i=0}^3 \left(\frac{\partial \eta_j}{\partial B} \psi_i^\dagger - \frac{\partial \chi_j}{\partial B} \psi_i \right) = 0, \\ \sum_{i=0}^3 \left(\frac{\partial \eta_j}{\partial C} \psi_i^\dagger - \frac{\partial \chi_j}{\partial C} \psi_i \right) = 0, \quad (21)$$

$$\alpha BC \psi_j^\dagger + \beta AC \psi_j^\dagger + \gamma AB \psi_j^\dagger + ABC \chi_j + ABC \sum_{i=0}^3 \left(\frac{\partial \eta_j}{\partial \psi_j} \psi_i^\dagger - \frac{\partial \chi_j}{\partial \psi_j} \psi_i \right) = 0, \quad (22)$$

$$\alpha BC \psi_j + \beta AC \psi_j + \gamma AB \psi_j + ABC \eta_j - ABC \sum_{i=0}^3 \left(\frac{\partial \eta_j}{\partial \psi_j^\dagger} \psi_i^\dagger - \frac{\partial \chi_j}{\partial \psi_j^\dagger} \psi_i \right) = 0, \quad (23)$$

$$(\alpha BC + \beta AC + \gamma AB) + ABC \frac{V'}{V} \sum_{i=0}^3 (\varepsilon_i \eta_i \psi_i^\dagger + \varepsilon_i \chi_i \psi_i) = 0. \quad (24)$$

Next, we will look for the solution in the following form

$$\alpha = N(A), \quad \beta = N(A) \cdot M(B), \quad \gamma = N(A) \cdot L(C), \quad (25)$$

and

$$\eta_j = N(A) \cdot Q(\psi_j), \quad \chi_j = N(A) \cdot P(\psi_j^\dagger). \quad (26)$$

From equation (22):

$$M = -\frac{B}{C} L. \quad (27)$$

Equations (23) and (24), if $L = C$, $M = -B$ will look like this:

$$A \frac{\partial N}{\partial A} + A^2 \frac{\partial N}{\partial A} - N = 0, \quad (28)$$

accordingly we get the following equation

$$N(A) = \frac{C_1 A}{A+1}, \quad (29)$$

To obtain a solution, we will consider the following form Q and P :

$$Q = -\left(\frac{1}{2} A^{-1} + \varepsilon_j \eta_0 \right) \psi_j, \quad P = -\left(\frac{1}{2} A^{-1} - \varepsilon_j \eta_0 \right) \psi_j^\dagger, \quad (30)$$

then

$$V = V_0 u, \quad (31)$$

$$F = F_0 u. \quad (32)$$

Substituting the last equation in the equation of motion (2), we find the value of T :

$$T = -\frac{V_0}{F_0} = \text{const.}$$

Exact cosmological solutions

Solutions will be sought in the form of de Sitter:

$$A = e^{\lambda_1 t}, B = e^{\lambda_2 t}, C = e^{\lambda_3 t},$$

we know that $\lambda = \lambda_1 = \lambda_2 = \lambda_3$, then from the equation of motion (5) we can determine the values

$\lambda = \mp \sqrt{\frac{V_0}{6F_0}}$, then energy density and pressur given as

$$\rho = \mp \sqrt{\frac{V_0}{2F_0}}, p = -3H^2 = -\frac{V_0}{2F_0}. \quad (33)$$

The equation of state parameter will look like this:

$$\omega = \frac{p}{\rho} = -1. \quad (34)$$

From equations (9) and (10) we obtain the following equation:

$$\dot{\Psi} + \left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} \right) \Psi = 0, \quad (35)$$

and integration gives

$$\Psi = \frac{\dot{\Psi}_0}{(ABC)^3}.$$

Conclusion

In this paper, we considered a model with a fermion field that is not minimally related to gravity within the framework of teleparallel gravity for Bianchi I type space-time. Here we defined the form of a point Lagrangian and obtained the corresponding field equations. To determine the bond forms and the potential function of the fermion field for the model under consideration, the Noether symmetry approach was used. For our model, we obtained of the coupling and fermionic field functions as $F = F_0 \Psi$ and $V = V_0 \Psi$. Then, we put these solutions to the field equations, we got the equation depend only on the functions A, B and C. After some mathematical calculations, we found the exact solutions for these functions as de Sitter solutions. We also determined that the equation of the state parameter is equal -1 . In

cosmology, this solution gives us a model of dark energy, which can describe the late epoch of evolution of our Universe. Thus, here the fermion fields play the role of dark energy.

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БЪЯНКИКЕҢІСТІК-УАҚЫТЫНДА ФЕРМИОНДЫ ӨРІСІ БАР КҮҢГІРТ ЭНЕРГИЯНЫҢ ТЕЛЕПАРАЛЛЕЛЬ МОДЕЛІ ҮШІН НЕТЕР СИММЕТРИЯ ӘДІСІ

Аннотация. Әлемнің үдемелі ұлғаюының табиғатын түсіндіре құрамды бөліктерді іздеуі космологияның іргелі тақырыбы болып табылады. Қарастырылып отырған жұмыста Бьянки I метрикасы үшін телепараллель гравитация аясында гравитациямен минималды емес әрекеттесетін фермионды өрістің модельі зерттеледі. Бұл жұмыста біз нүктелік лагранжианның түрін анықтадық. Қарастырылып отырған модель үшін фермионды өрістің потенциалының және байланыс функциясының түрін анықтауда Нетер симметриясы әдісін пайдаландық. Қазіргі таңда космологияда аталған әдісті – зерттеліп отырған модель үшін белгісіз функцияны анықтауға және нақты космологиялық шешімдерді алуда жиі қолданады. Өзіміздің моделіміз үшін фермионды өрістің байланыс функциясын алдық. Содан кейін алынған шешімдерді өріс теңдеулеріне қойып, А, В және С функцияларына ғана тәуелді теңдеуді аламыз. Математикалық есептеулерден соң осы функциялар үшін де-Ситтер түріндегі нақты шешімдерді таптық. Нәтижесінде күй теңдеуінің мәні – 1 екендігін анықтадық. Космологияда бұл шешім күнгірт энергияның моделін береді және біздің Әлем эволюциясының кеш дәуірін сипаттай алады. Сонымен біздің жұмысымызда анықталғаны – фермионды өрістер күнгірт энергияның рөлін ойнайды.

Түйін сөздер. Телепараллельді күнгірт энергия, Бьянки I типті моделі, фермионды өріс, Нетер симметрия әдісі.

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ПОДХОД НЕТЕРОВОЙ СИММЕТРИИ В ТЕЛЕПАРАЛЛЕЛЬНОЙ МОДЕЛИ ТЕМНОЙ ЭНЕРГИИ С ФЕРМИОННЫМ ПОЛЕМ ДЛЯ ПРОСТРАНСТВА-ВРЕМЕНИ ТИПА I БЪЯНКИ

Аннотация. Поиск составных частей, которые могут объяснить периоды ускоряющегося расширения Вселенной является фундаментальной темой космологии. В настоящей работе рассматривается модель с фермионным полем, не минимально связанным с гравитацией в рамках телепараллельной гравитации для пространства-времени типа Бьянки I. Здесь мы определили форму точечного лагранжиана и получили соответствующие уравнения поля. Для определения форм связи и потенциальной функции фермионного поля для рассматриваемой модели используется подход симметрии Нетер. В современной космологии часто используется этот подход для определения неизвестной функции и получения точных космологических решений для рассматриваемой модели. Для нашей модели мы получили функции связи и фермионного поля. Затем мы подставляем эти решения в уравнения поля, мы получаем уравнение, зависящее только от функций А, В и С. После некоторых математических вычислений мы нашли точные решения для этих функций как решения де Ситтера. Наконец, мы определили, что уравнение параметра состояния равно -1. В космологии это решение дает нам модель темной энергии, которая может описать позднюю эпоху эволюции нашей Вселенной. Таким образом, здесь фермионные поля играют роль темной энергии.

Ключевые слова: телепараллельная темная энергия, модель Бьянки типа I, фермионное поле, подход симметрии Нетер.

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